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Abstract

We propose a novel theory of financial contagion. We study global coordination games of regime change in two regions with an initially uncertain correlation of regional fundamentals. A crisis in region 1 is a wake-up call to investors in region 2 that induces a reassessment of local fundamentals. Contagion after a wake-up call can occur even if investors learn that fundamentals are uncorrelated and common lender effects or balance-sheet linkages are absent. Applicable to currency attacks, bank runs and debt crises, our theory of contagion is supported by existing evidence and generates a new testable implication for empirical work.

JEL classification: D82, F3, G01
Bank classification: Exchange rates; Financial stability; International financial markets

Résumé

Les auteurs proposent une théorie inédite de la contagion financière. Ils étudient les jeux globaux de coordination autour d'un changement de régime dans deux régions dont les facteurs fondamentaux présentent initialement une corrélation incertaine. Une crise dans la première région envoie un signal d'alerte aux investisseurs de la seconde, ce qui pousse ces derniers à réévaluer les facteurs fondamentaux de leur propre région. Il peut y avoir contagion après l'envoi de ce signal, même si les investisseurs apprennent que les facteurs fondamentaux des deux régions ne sont pas corrélés et qu'on observe une absence d'effets de créancier commun ou de liens entre les bilans. Cette théorie de la contagion, qui s'applique aux attaques contre la monnaie, aux retraits massifs de dépôts bancaires et aux crises d'endettement, est corroborée par les données existantes et génère une nouvelle prédiction susceptible d'être testée par des travaux empiriques.

Classification JEL : D82, F3, G01
Classification de la Banque : Taux de change; Stabilité financière; Marchés financiers internationaux

Non-Technical Summary

What are the causes and channels of financial contagion? These questions are important for academics and policy-makers alike. One popular explanation for financial contagion is the wake-up-call hypothesis. According to this hypothesis, a financial crisis in region 1 is a wake-up call to investors in region 2 that induces them to reassess the fundamentals of region 2. Such a reappraisal of risk can lead to a financial crisis in region 2, either due to weaker local fundamentals (perhaps because of exposure to region 1), or due to greater uncertainty about local fundamentals.

There is a great deal of empirical evidence on contagion based on wake-up calls across markets and over time. Studying equity markets during the global financial crisis of 2007–09, wake-up calls were identified as the key driver of contagion. Analyzing euro area sovereign bond markets, contagion based on the wake-up call of the Greek crisis of 2009–10 has been documented. Studying bond markets during the Asian crisis in 1997, evidence for contagion based on the reassessment of risks in some countries has been reported. The Russian crisis in 1998 has been viewed as the outcome of a wake-up call in emerging markets. Furthermore, wake-up call effects have been documented in banking crises, such as during the Russian banking panic of 2004 and the Panic of 1893 in the United States.

Despite the empirical evidence for the wake-up-call contagion channel, there has been little theoretical work on the wake-up-call hypothesis. Our paper closes this gap by proposing a wake-up-call theory of contagion. We focus on the coordination aspect of financial crises, which is at the heart of currency crises, bank runs and debt crises. A crisis occurs if sufficiently many depositors withdraw from a bank, currency speculators attack a peg or creditors do not roll over debt.

Based on the global games framework, we develop a model with two regions that move sequentially and where the correlation of regional fundamentals is uncertain ex ante. Contagion is defined to occur when the probability of a crisis in region 2 is higher after a crisis in region 1 than after no crisis in region 1.

We show that contagion occurs even if investors learn that regional fundamentals are uncorrelated ex post and when common lenders or balance-sheet links are absent. Thus, our theory explains how a wake-up call *in isolation* transmits financial crises. It thereby captures the *wake-up-call component* of contagion.

1 Introduction

The causes of financial contagion are an important question in international finance. For example, Forbes (2012) distinguishes four mutually non-exclusive channels of contagion: trade, banks, portfolio investors and wake-up calls. According to the wake-up-call hypothesis, a popular explanation for contagion put forward by Goldstein (1998), a financial crisis in region 1 is a wake-up call to investors in region 2 that induces them to reassess the fundamentals of region 2. Such a reappraisal of risk can lead to a financial crisis in region 2, either due to weaker local fundamentals or greater uncertainty about local fundamentals.

The empirical literature documents support for wake-up-call contagion across markets and over time. Studying equity markets during the global financial crisis of 2007–09, Bekaert et al. (2014) identify wake-up calls as the key driver of contagion. Analyzing euro area sovereign bond markets, Giordano et al. (2013) find empirical evidence for contagion based on the wake-up call of the Greek crisis of 2009–10. Studying bond markets during the Asian crisis in 1997, Basu (2002) finds evidence for contagion based on the reassessment of risks in some countries. Van Rijckeghem and Weder (2003) view the Russian crisis in 1998 as the outcome of a wake-up call in emerging markets. Karas et al. (2013) find a wake-up-call effect during the Russian banking panic of 2004, where the sensitivity of deposit flows to bank capital remained elevated, regardless of the introduction of deposit insurance. From a historical perspective, Ramirez and Zandbergen (2013) document evidence for contagion based on the wake-up call of newspaper reports about distant bank runs in the Panic of 1893, which led to elevated deposit withdrawals in Montana.

Despite the empirical evidence, there has been little theoretical work on the wake-up-call hypothesis. Our paper closes this gap by proposing a wake-up-call theory of contagion. Based on global games (Carlsson and van Damme 1993), we develop a model with two regions that move sequentially and where the correlation of regional fundamentals is uncertain ex ante. Contagion is defined to occur when the probability of a crisis in region 2 is higher after a crisis in region 1 than after no crisis in region 1. We show that contagion occurs even if investors learn that regional fundamentals are uncorrelated ex post and when common lenders or balance-sheet links are absent. Our theory explains how a wake-up call *in isolation* transmits financial crises. It captures the *wake-up-call component* of contagion.

We consider a standard global coordination game of regime change with incomplete information about the fundamental (Morris and Shin 2003) in each region. A financial crisis occurs in a given region if sufficiently many investors act against the regime (attack a currency peg, withdraw funds from a bank, or refuse to roll over debt). In contrast to the standard game, our model is sequential. First, investors in region 1 decide whether to act, which determines the outcome in region 1. Afterwards, investors in region 2 observe whether a regime change occurred in region 1. Moreover, if and only if a crisis occurs in region 1, additional information becomes available to investors in region 2, or can be acquired cheaply. Specifically, after a crisis in region 1, all investors in region 2 observe the fundamental in region 1, and a proportion of investors observe the realized correlation of regional fundamentals. This informational asymmetry around crises is a key assumption that can be justified by the news coverage of crises and public inquiries. We discuss in section 5.3 how this assumption can be relaxed without affecting our key insights.

We start by analyzing the case of exogenous information. Since investors are heterogeneously informed about the correlation of fundamentals, the prior beliefs about the fundamental of region 2 are heterogeneous. We show that a unique equilibrium in region 2 exists in this environment if private information about the fundamental is sufficiently precise (Proposition 1).

A crisis in region 1 is a wake-up call to investors in region 2. Our main result is to show that contagion can occur even if these investors learn that fundamentals are uncorrelated (Proposition 2). By focusing on the case in which fundamentals are observed to be uncorrelated, we isolate the wake-up-call component of contagion and, hence, go beyond information contagion.

The wake-up call induces investors to reassess the fundamental in region 2 in two ways. First, the mean of the local fundamental is lower after the wake-up call. No crisis in region 1 would have been favorable news for region 2, since fundamentals may be positively correlated, resulting in a positive *mean effect*. In contrast, learning that fundamentals are uncorrelated after a crisis in region 1 has an overall neutral effect, since information about region 1 is uninformative, resulting in no effect on the mean of the local fundamental in region 2. Taken together, the probability of a crisis in region 2 is higher after a crisis in region 1 and learning about uncorrelated fundamentals than after no crisis in region 1.

Second, the variance of the local fundamental is higher after a wake-up call. When fundamentals are known to be uncorrelated, observing a crisis in region 1 is uninformative for investors in region 2. Hence, there can be greater disagreement among investors about the fundamental in region 2. Since public information about the local fundamental is less precise, investors who are informed about the zero correlation put greater weight on their dispersed private information. Greater disagreement is reflected in more-dispersed forecasts. This *variance effect* can increase the probability of a crisis in region 2 (Metz 2002; Heinemann and Illing 2002). As a result, investors attack the regime more aggressively.

Both the mean and the variance effects go in the *same* direction for the result of wake-up-call contagion. The variance effect is absent in the special case where all investors are uninformed about the zero correlation of fundamentals. Wake-up-call contagion still obtains, since the mean effect in isolation suffices for the result.

We further explore the effect of disagreement among investors on contagion. We show that, if fundamentals are uncorrelated, contagion can *increase* in the proportion of informed investors (Proposition 3). As more investors are informed, more investors learn that fundamentals are uncorrelated, thus revising upward both the mean and the variance of the local fundamental. In this case, the mean and variance effects go in *opposite* directions. This result on an *enhanced perception of risk* hinges on a large variance effect, which enhances the disagreement among informed investors. Specifically, for the variance effect to outweigh the mean effect, a lower bound on the fundamental in region 1 is required.

Our result on the enhanced perception of risk has new implications for the empirical literature on banking and currency crises. This literature studies the role of trade links (Glick and Rose 1999), financial links (Van Rijckeghem and Weder 2001, 2003), and institutional similarities (Dasgupta et al. 2011). Our theory suggests that the likelihood of contagion depends non-linearly on the characteristics of region 1. In particular, after controlling for the fundamentals of region 2, a crisis in region 1 due to extremely low fundamentals is less likely to spread if fundamentals are uncorrelated. Conversely, a crisis in region 1 due to moderately low fundamentals is more likely to spread if fundamentals are uncorrelated.[1]

[1]The importance of non-linearities has been examined by Forbes and Rigobon (2002) and Bekaert et al. (2014) in the context of financial market returns and the transmission of information. Favero and Giavazzi (2002) contrast contagion with "flight-to-quality" episodes.

Building on a standard global coordination game of regime change, the wake-up-call theory of contagion has several applications.[2] For currency crises, speculators observe a currency attack and are uncertain about the magnitude of trade or financial links or institutional similarity.[3] For rollover risk and bank runs, wholesale investors observe a run elsewhere and are uncertain about interbank exposures.[4] For sovereign debt crises, bond holders observe a sovereign default elsewhere and are uncertain about the macroeconomic links, the commitment of the international lender of last resort, or the resources of multilateral bailout funds.[5] For political regime change, activists observe a revolution, e.g. during the Arab Spring, and are uncertain about the impact on their government's ability to stay in power.[6]

In contrast to alternative theories of contagion, we demonstrate that contagion can occur even if common lender effects or balance-sheet linkages are absent and investors learn that fundamentals are uncorrelated to the crisis region ex post. Regarding balance-sheet links, see Allen and Gale (2000) and Dasgupta (2004) for interbank links and Kiyotaki and Moore (2002) for balance-sheet contagion. For a common discount factor channel, see Ammer and Mei (1996) and Kodres and Pritsker (2002). Regarding a common investor base, see Goldstein and Pauzner (2004) for risk aversion, Pavlova and Rigobon (2008) for portfolio constraints, and Taketa (2004) and Oh (2013) for learning about other investors. In terms of ex-post correlated fundamentals, see Basu (1998) for a common risk factor, and Acharya and Yorulmazer (2008) and Allen et al. (2012) for asset commonality among banks and information contagion. See Chen (1999) for a model with information contagion and uninformed junior claimants. See Chen and Suen (2013) for a model of information contagion in the context of model uncertainty. In contrast, we provide a novel and complementary theory of contagion based on the reassessment of local fundamentals after a wake-up call.[7]

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[2]See also Angeletos et al. (2006) and Dasgupta (2007).

[3]See also Morris and Shin (1998) and Corsetti et al. (2004) for a one-regional global game that builds on the earlier work of Krugman (1979), Flood and Garber (1984), and Obstfeld (1986).

[4]See also Rochet and Vives (2004) and Goldstein and Pauzner (2005) for a one-regional global game that builds on the earlier work of Diamond and Dybvig (1983).

[5]See also Corsetti et al. (2006). See Drazen (1999) for membership contagion.

[6]For a one-regional global game of political regime change with endogenous information manipulation or dissemination, see Edmond (2013) and Shadmehr and Bernhardt (2015), respectively.

[7]Contagion arises in Calvo and Mendoza (2000), since globalization shifts the incentives of investors from costly information acquisition to imitation and detrimental herding. By contrast, contagion arises in our paper because investors acquire information after a wake-up call, which induces the reassessment of local fundamentals.

We show that our contagion results and the underlying properties of equilibria prevail under endogenous information, provided that information is more cheaply available after a crisis. We start by analyzing the value of information about the correlation of regional fundamentals. In the model with *exogenous information*, an exogenous proportion of investors learn about the correlation after a crisis in region 1. We find that the private value of information about the correlation of fundamentals increases in the proportion of informed investors. This strategic complementarity in information choices is similar to Hellwig and Veldkamp (2009), who first studied the optimal information choice in strategic models. They show that the information choices of investors inherit the strategic motive of the underlying beauty contest game. In contrast, we study the acquisition of publicly available information about the correlation of fundamentals in a global game of regime change.

In our model, the priors about the regional fundamental are heterogeneous across investors. This arises from both the initial uncertainty about the correlation and the learning of the realized correlation by a proportion of investors. Specifically, the prior of uninformed investors follows a mixture distribution.[8] Information about the correlation can increase or decrease the precision of the prior about the local fundamental. Hence, there can be greater disagreement among informed investors after a wake-up call, which contributes to contagion. While uninformed investors play an invariant strategy, informed investors can tailor their strategy to the observed correlation. This benefit of tailoring their strategy to the observed correlation underpins the strategic complementarity in information choice and, in the model with *endogenous information*, provides incentives for investors to acquire costly information about the correlation.

Based on the strategic complementarity in information choices, we find that there exists an equilibrium in which all investors acquire information after a crisis in region 1. This allows us to reinterpret our previous contagion results for the case of endogenous information, since investors do not acquire information after no crisis in region 1, provided information is more cheaply available after a crisis event. In section 5.3, we discuss how the assumption of the informational asymmetry (the availability or cost of information depending on a whether a crisis occurred in region 1) can be relaxed without affecting our key insights. Finally, we also endo-

[8] In a global game with mixture distributions, Chen et al. (2012) develop a theory of rumors during political regime change. However, they abstract from both contagion and information choice.

genize the information precision of private signals, as in Szkup and Trevino (2012), and show that private information choice strengthens our results.

This paper proceeds as follows. We describe our global games model with initial uncertainty about the correlation of fundamentals in section 2. Using mixture distributions, we obtain the unique equilibrium under exogenous information in section 3. Next, we establish the result of contagion after a wake-up call under exogenous information. In section 4, we develop the additional result of the enhanced perception of risk after a wake-up call and derive a testable implication. Subsequently, we endogenize the information of investors in section 5, where we also establish a strategic complementarity in information choices. Furthermore, we discuss a relaxation of our assumption about the information asymmetry and show how our results are enhanced when investors can also acquire more-precise private information. Section 6 concludes. Derivations and proofs are in the appendices.

2 Model

We study a sequence of global coordination games of regime change in two regions indexed by $t \in \{1,2\}$. Each region is inhabited by a different unit continuum of risk-neutral investors indexed by $i \in [0,1]$. Investors in region $t=1$ move first and are followed by investors in region $t=2$.

In each region, investors simultaneously decide whether to attack the regime, $a_{it}=1$, or not, $a_{it}=0$. The outcome of the attack depends on both the aggregate attack size, $A_t \equiv \int_0^1 a_{it}\,di$, and a regional fundamental $\theta_t \in \mathbb{R}$ that measures the strength of the regime. A regime change occurs if sufficiently many investors attack, $A_t > \theta_t$. Following Vives (2005), an attacking investor receives a benefit $b_t > 0$ if a regime change occurs, and incurs a loss $\ell_t > 0$ otherwise, where $\gamma_t \equiv \frac{\ell_t}{b_t+\ell_t} \in (0,1)$ captures an investor's relative cost of failure in region t (investor conservatism):

$$u(a_{it}=1, A_t, \theta_t) = b_t \;\mathbf{1}_{\{A_t>\theta_t\}} - \ell_t \;\mathbf{1}_{\{A_t\leq\theta_t\}}. \tag{1}$$

The payoff from not attacking is normalized to zero. Thus, the relative payoff from attacking increases in the attack size A_t and decreases in the fundamental θ_t. Hence, the attack decisions of investors exhibit global strategic complementarity.

A regime change can be a currency crisis, a bank run or a sovereign debt crisis. The fundamental can be interpreted as the ability of a monetary authority to defend its currency (Morris and Shin 1998; Corsetti et al. 2004), the measure of investment profitability (Rochet and Vives 2004; Goldstein and Pauzner 2005; Corsetti et al. 2006), or a sovereign's taxation power or willingness to repay. Investors are interpreted as currency speculators, retail or wholesale bank creditors who withdraw funds, or sovereign debt holders who refuse to roll over.

The first key feature of our model is an initial uncertainty about the correlation between regional fundamentals. We assume that the correlation $\rho \equiv corr(\theta_1, \theta_2)$ is zero with probability $p \in (0,1)$ or takes a positive value $\rho_H \in (0,1)$:

$$\rho = \begin{cases} 0 & \text{w.p. } p \\ \rho_H & \text{w.p. } 1-p. \end{cases} \tag{2}$$

The initial uncertainty about the correlation of regional fundamentals is motivated by our applications to financial crises. In the context of currency attacks, the ex-ante uncertain correlation reflects the unknown magnitude of trade or financial links or the unknown institutional similarity. In the context of bank runs, it reflects the uncertainty about interbank exposures. In the context of sovereign debt crises, the uncertain correlation reflects the uncertainty about the macroeconomic and financial links across countries. It could also reflect the uncertainty about the resources and commitment of multilateral bailout funds or the international lender of last resort.

Fundamentals follow a bivariate normal distribution with mean $\mu_t \equiv \mu$, precision $\alpha_t \equiv \alpha \in (0,\infty)$, and realized correlation ρ. There is incomplete information about the fundamental θ_t (Carlsson and van Damme 1993). Each investor receives a noisy private signal x_{it} before the attack decision (Morris and Shin 2003):

$$x_{it} \equiv \theta_t + \varepsilon_{it}, \tag{3}$$

where idiosyncratic noise ε_{it} is identically and independently normally distributed across investors with zero mean and precision $\beta \in (0,\infty)$. The regional fundamentals, the correlation and the sequences of idiosyncratic noise terms are independent.

The second key feature is an informational asymmetry. Only after a financial crisis in region 1, additional information becomes available to investors in region 2,

or can be acquired cheaply by them. This assumption can be justified by the news coverage of crises and public inquiries. Therefore, we assume that the realized fundamental θ_1 is publicly observed and a proportion $n \in [0,1]$ of investors learn the realized correlation ρ. These two pieces of additional information are available if and only if a crisis occurs in region 1. We discuss the relaxation of this informational asymmetry in section 5.3. The information structure is common knowledge.

Table 1 summarizes the timeline of events.

Date 1:

- The correlation of fundamentals ρ is realized but unobserved.
- The fundamentals (θ_1, θ_2) are realized but unobserved.

Coordination stage

- Investors receive private information x_{i1} about the fundamental θ_1.
- Investors simultaneously decide whether to attack a_{i1}.
- Payoffs to investors in region 1.

Date 2: **Information stage: fundamental reassessment in region** 2

- After a crisis in region 1, the following information is available:
  - the fundamental θ_1 is observed and
  - a proportion n of investors obtain information about the correlation ρ.
- In contrast, θ_1 and ρ are unobserved if there is no crisis in region 1.
- Investors reassess the local fundamental θ_2.

Coordination stage

- Investors receive private information x_{i2} about the fundamental θ_2.
- Investors simultaneously decide whether to attack a_{i2}.
- Payoffs to investors in region 2.

Table 1: Timeline of events

3 Unique equilibrium with wake-up-call contagion

We briefly review the well-known equilibrium in region 1 (e.g., Morris and Shin 2003). A Bayesian equilibrium is an attack decision a_{i1} for each investor i and an aggregate attack size A_1 that satisfy both individual optimality and aggregation:

$$\begin{aligned} a_{i1}^* &= \arg \max_{a_{i1} \in \{0,1\}} \mathbb{E}[u(a_{i1}, A_1, \theta_1) | x_{i1}] \equiv a(x_{i1}), \ \forall i \\ A_1^* &= \int_{-\infty}^{+\infty} a(x_{i1}) \sqrt{\beta} \phi(\sqrt{\beta}(x_{i1} - \theta_1)) dx_{i1} \equiv A(\theta_1), \end{aligned}$$

where $\phi(\cdot)$ and $\Phi(\cdot)$ denote the probability density function (pdf) and cumulative density function (cdf) of the standard Gaussian.

Result 1 ***Morris and Shin (2003)*** *If private information is sufficiently precise, $\beta > \underline{\beta}_0 \equiv \frac{\alpha^2}{2\pi} \in (0, \infty)$, then there exists a unique Bayesian equilibrium in region 1. This equilibrium is characterized by a signal threshold, x_1^*, and a fundamental threshold, θ_1^*. Investor i attacks whenever $x_{i1} < x_1^*$, and a crisis occurs whenever $\theta_1 < \theta_1^*$. The fundamental threshold θ_1^* is defined by*

$$F_1(\theta_1^*) \equiv \Phi\left(\frac{\alpha}{\sqrt{\alpha + \beta}} (\theta_1^* - \mu) - \sqrt{\frac{\beta}{\alpha + \beta}} \Phi^{-1}(\theta_1^*) \right) = \gamma_1, \tag{4}$$

and the signal threshold x_1^* is defined by $x_1^* = \theta_1^* + \frac{1}{\sqrt{\beta}} \Phi^{-1}(\theta_1^*)$.

To simplify the exposition, we make an assumption to ensure that $\theta_1^* = \mu$:

$$\gamma_1 \equiv 1 - \Phi\left(\frac{\beta}{\sqrt{\alpha + \beta}} \Phi^{-1}(\mu) \right). \tag{5}$$

The fundamental threshold θ_1^* decreases in μ and increases in γ_1. Therefore, there exists a unique γ_1 that ensures that $\theta_1^* = \mu$. Our results generalize, as shown in Ahnert and Bertsch (2013).

We next turn to region 2. Investors in region 2 use all available information to reassess the local fundamental θ_2. If no crisis occurred in region 1, investors only learn the event $\theta_1 \geq \theta_1^*$, but not the realized θ_1. If a crisis occurred in region 1 – *a wake-up call* – more information is available to investors in region 2. All investors learn θ_1 and a proportion n of investors are informed, learning the correlation ρ.

The equilibrium in region 2 is characterized by indifference and critical mass conditions. Different from the analysis of region 1, there are two distinct fundamental thresholds – one for each realized correlation – and thus two critical mass conditions. Similarly, there are three indifference conditions – one for uninformed investors and one for informed investors for each realized correlation. We derive these conditions in Appendix A.1.

Proposition 1 ***Existence of a unique monotone equilibrium.*** *If private information is sufficiently precise, there exists a unique monotone Bayesian equilibrium in region 2 for any proportion of informed investors, $n \in [0,1]$. This equilibrium is characterized by signal thresholds for informed investors, $x_I^*(n,\rho,\theta_1)$, and for uninformed investors, $x_U^*(n,\theta_1)$, as well as a fundamental threshold $\theta_2^*(n,\rho,\theta_1)$ for each realized correlation $\rho \in \{0,\rho_H\}$. Investors attack whenever their private signal is sufficiently low, $x_{i2} < x_U^*(n,\theta_1)$ if uninformed and $x_{i2} < x_I^*(n,\rho,\theta_1)$ if informed. A crisis occurs whenever the fundamental is sufficiently low, $\theta_2 < \theta_2^*(n,\rho,\theta_1)$.*

Proof See Appendix A.2, in which we also derive the thresholds.

The equilibrium analysis of region 2 is more complicated for two reasons. First, after a crisis in region 1, the priors about the regional fundamental are heterogeneous across investors, since only informed investors observe the correlation. Second, both the prior and the posterior of an uninformed investor follow a mixture distribution, so normality is lost. Similarly, mixture distributions are used for all investors after no crisis in region 1. Using the results of Milgrom (1981) and Vives (2005), we show that the best-response function of an individual investor strictly increases in the thresholds used by other investors (see also Appendix A.1.3). Hence, the common requirement of precise private information suffices for uniqueness in monotone equilibrium despite heterogeneous priors and mixture distributions.

After the wake-up call of a crisis in region 1, *informed investors* reassess the fundamental in region 2 by using both θ_1 and ρ. They form an updated prior, where normality is preserved. The conditional mean is $\mu_2|\rho,\theta_1 = \rho\theta_1 + (1-\rho)\mu \equiv \mu_2(\rho,\theta_1)$, and the conditional variance is $\alpha_2|\rho = \frac{\alpha}{1-\rho^2} \equiv \alpha_2(\rho)$:

$$\theta_2|\rho,\theta_1 \sim \mathcal{N}\left(\rho\theta_1 + (1-\rho)\mu, \frac{1-\rho^2}{\alpha}\right). \tag{6}$$

By contrast, *uninformed investors* can only use θ_1 to reassess the fundamental in region 2. They form a mixture distribution between $\theta_2|\rho = 0$ and $\theta_2|\rho = \rho_H, \theta_1$, using the ex-ante distribution of the correlation as weights:

$$\theta_2|\theta_1 \equiv p \cdot [\theta_2|\rho = 0] + (1-p) \cdot [\theta_2|\rho = \rho_H, \theta_1]. \tag{7}$$

Similarly, after no crisis in region 1, all investors are uninformed and build a weighted average over this mixture distribution for all $\theta_1 \geq \theta_1^*$.

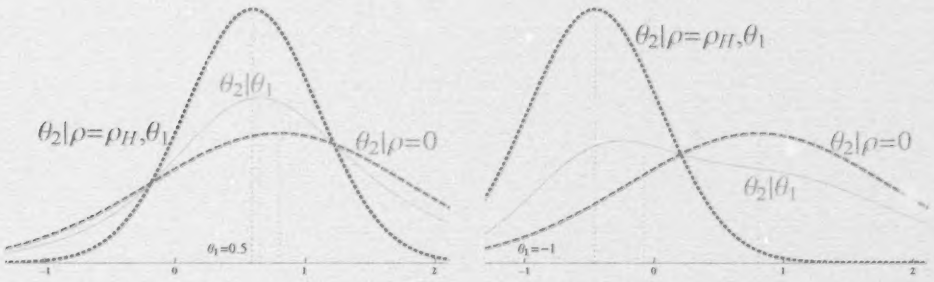

Figure 1: Reassessment of the local fundamental: The updated prior distributions of informed investors for zero correlation (dashed brown), positive correlation (dotted blue) and of uninformed investors (solid red). Parameters: $\mu = 0.8$, $\alpha = 1$, $p = 0.7$, $\rho_H = 0.7$, $\theta_1 = 0.5$ (left panel), $\theta_1 = -1$ (right panel).

Figure 1 shows the reassessment of the local fundamental after a crisis in region 1. It depicts the updated prior distributions of investors. The updated prior of informed investors, who learn about a zero correlation, has the highest mean and variance. In contrast, learning about a positive correlation leads to an updated prior distribution with the lowest mean and variance. The updated prior distribution of uninformed investors can be unimodal, similar to a normal distribution with fat tails (left panel), or bimodal for small values of θ_1 (right panel).

Subsequently, investors use their private information x_{i2} to form a posterior about the fundamental in region 2. First, the posterior of informed investors depends on the correlation, $\theta_2|\rho = 0, x_{i2}$ and $\theta_2|\rho = \rho_H, x_{i2}$. These posterior distributions are conditionally normally distributed with greater precision and a mean shifted toward the private signal x_{i2}. Second, uninformed investors do not observe the realized correlation, so they form a belief using the observed fundamental, θ_1, and the private signal about the fundamental in region 2, x_{i2}. Let $\hat{p}$ denote this belief

about a zero correlation of fundamentals derived and analyzed in Appendix A.1.2:

$$\hat{p} \equiv \Pr\{\rho = 0 | \theta_1, x_{i2}\}. \tag{8}$$

Using the updated belief $\hat{p}$ as a weight, the posterior about θ_2 is again an average over the cases of positive and zero correlation, which follows a mixture distribution:

$$\theta_2 | \theta_1, x_{i2} \equiv \hat{p} \cdot [\theta_2 | \rho = 0, x_{i2}] + (1 - \hat{p}) \cdot [\theta_2 | \rho = \rho_{II}, \theta_1, x_{i2}]. \tag{9}$$

The case of no crisis in region 1 is similar, since all investors are uninformed about the correlation of fundamentals. They build a weighted average over this mixture distribution for all $\theta_1 \geq \theta_1^*$ (see also the proof of Proposition 1).

Having established the existence of unique equilibrium, we next derive a result about information contagion in our set-up with uncertain correlation. A crisis in region 1 is unfavorable news about the fundamental in region 1. Since fundamentals may be correlated, this crisis may also be unfavorable news about the fundamental in region 2. The reassessment of the local fundamental θ_2 reduces the expected fundamental, increasing the probability of a crisis in region 2.

This result mirrors the existing literature on contagion due to ex-post correlated fundamentals. Information contagion has been established by Acharya and Yorulmazer (2008) and Allen et al. (2012). Acharya and Yorulmazer (2008) show that the funding cost of one bank increases after bad news about another bank when the banks' loan portfolio returns have a common factor. To avoid information contagion ex post, banks herd their investment ex ante. Allen et al. (2012) compare the impact of information contagion on systemic risk across asset structures. Adverse news about the solvency of the banking system leads to runs on multiple banks.

Lemma 1 ***Information contagion.*** *If private information is sufficiently precise and investors are uninformed about the correlation of fundamentals, $n = 0$, then a crisis in region 2 is more likely after a crisis in region 1 than after no crisis:*

$$\Pr\{\theta_2 < \theta_2^*(0, \rho, \theta_1) \mid \theta_1 < \theta_1^*\} > \Pr\{\theta_2 < \theta_2^*(0, \rho, \theta_1) \mid \theta_1 \geq \theta_1^*\}. \tag{10}$$

Proof See Appendix B.1.

This information contagion result obtains for the case in which all investors are uninformed. Lemma 1 compares the probability of a crisis in region 2 conditional on whether a crisis occurred in region 1. At the core of the result is a *mean effect*. The prior about the fundamental is less favorable when observing a crisis in region 1 due to the potentially positive fundamental correlation. Information contagion is consistent with the empirical findings of Eichengreen et al. (1996), whereby a currency crisis elsewhere increases the probability "of a speculative attack by an economically and statistically significant amount" (p. 2). Notably, the result of Lemma 1 can be further generalized for all $n \in [0,1)$.

Next, we demonstrate that contagion can occur even if investors learn that regional fundamentals are uncorrelated. Thereby, we go beyond information contagion, where all investors are uninformed. We show that contagion can occur after a wake-up call even if investors are informed and learn that regions are unrelated. Thus, contagion can occur when fundamentals are uncorrelated ex post and there is no common investor base or balance-sheet link. Therefore, we call the result wake-up-call contagion in isolation. Before stating this result formally in Proposition 2, we need to characterize the strength of fundamentals.

Definition 1 ***Strong prior.*** *The prior about the fundamental is strong if* $\mu_2(\rho,\theta_1) > \max\{X(\rho),Y(\rho)\}$ *for each realized correlation* $\rho \in \{0,\rho_H\}$*, where:*

$$X(\rho) \equiv \Phi\left(-\frac{\sqrt{\alpha_2(\rho)+\beta}}{\sqrt{\beta}}\Phi^{-1}(\gamma_2)\right), \tag{11}$$

$$Y(\rho) \equiv \frac{1}{2}-\frac{\sqrt{\alpha_2(\rho)+\beta}}{\alpha_2(\rho)}\Phi^{-1}(\gamma_2). \tag{12}$$

We focus on a strong prior, which shifts interest to the left tail of the distribution, and investors attack only after receiving a relatively low signal. As shown in Appendix B.2, a weak prior makes a crisis more likely relative to the prior, $\mu_2(\rho,\theta_1) < \theta_2^*(1,\rho,\theta_1) < 1$, while a strong prior makes a crisis relatively less likely, $0 < \theta_2^*(1,\rho,\theta_1) < \mu_2(\rho,\theta_1)$, for each realized correlation. Furthermore, the accompanying comparative statics for a strong prior are $d\theta_2^*(1,\rho,\theta_1)/d\alpha < 0$, $d\theta_2^*(1,\rho,\theta_1)/d\beta > 0$ and $d\theta_2^*(1,\rho,\theta_1)/d\mu_2 < 0$. Absent ex-ante uncertainty and learning, similar comparative statics have been derived by Metz (2002). Equipped with Definition 1, a main result under exogenous information follows.

Proposition 2 ***Wake-up-call contagion in isolation.*** *Suppose private information is sufficiently precise, public information is sufficiently imprecise, and the prior is strong. Then, a crisis in region* 2 *is more likely after a crisis in region* 1 *— even if a proportion* $n \in (0, 1]$ *of investors learn that fundamentals are uncorrelated:*

$$\Pr\{\theta_2 < \theta_2^*(n, \rho, \theta_1) | \rho = 0, \theta_1 < \theta_1^*\} > \Pr\{\theta_2 < \theta_2^*(0, \rho, \theta_1) | \theta_1 \geq \theta_1^*\}. \quad (13)$$

Proof See Appendix B.3.

The right-hand side of inequality (13) is unchanged relative to Lemma 1 and represents the probability of a crisis in region 2 after no crisis occurred in region 1. Since the correlation is unobserved in this contingency, the conditional probability allows for any realization of the correlation $\rho \in \{0, \rho_H\}$. By contrast, the left-hand side of inequality (13) is the probability of a crisis in region 2 after a crisis occurred in region 1 *and* fundamentals are uncorrelated. A positive proportion of investors, or even all investors, learn about the zero correlation of fundamentals.

Intuition If fundamentals are uncorrelated, a crisis in region 1 does not affect the probability of a crisis in region 2. If fundamentals are correlated, however, a crisis in region 1 has consequences for contagion. Specifically, the conditional probabilities on both sides of inequality (13) differ for two reasons, each associated with the reassessment of the local fundamental θ_2.

First, the mean of the local fundamental matters. Learning that no crisis occurred in region 1 (favorable news about θ_2) improves the mean of the updated prior on the right-hand side because θ_1 and ρ are not observed and regional fundamentals are potentially positively correlated. In contrast, after a crisis occurred in region 1, learning that fundamentals are uncorrelated also improves the mean of the updated prior on the left-hand side, but by less. This is because the favorable news of $\rho = 0$ after the unfavorable news of a crisis in region 1 has an overall neutral effect on the mean of θ_2. Hence, this *mean effect* works toward the inequality stated in Proposition 2.

Second, the variance of the local fundamental matters. On the left-hand side, the public information about the local fundamental θ_2 is less precise after learning that fundamentals are uncorrelated. Consequently, private information becomes

relatively more precise, which results in greater disagreement among informed investors, who learn that fundamentals are uncorrelated. If the prior is strong, greater disagreement translates into more-aggressive attacks and a larger probability of a crisis (Metz 2002; Heinemann and Illing 2002). This *variance effect* works toward the inequality stated in Proposition 2.

We stress that both the mean and the variance effects go in the *same* direction. If no investor is informed, $n = 0$, the variance effect is zero. Hence, the mean effect suffices to obtain the result in Proposition 2. If some investors are informed after a wake-up call, however, the variance effect also contributes to inequality (13).

Relation to the empirical literature In the empirical literature, wake-up-call contagion is often captured as a subset of information contagion, after having controlled for various alternative channels of contagion. The objective of our theory is to isolate the wake-up-call component of contagion, which is achieved by focusing on the element of contagion that prevails even if investors learn that fundamentals are uncorrelated. Translated to the empirical literature, we focus on the effect over and above the (fundamental) contagion due to an observed correlation of fundamentals ex post. In particular, after accounting for the fundamentals of region 1 and 2, there remains an interaction between the occurrence of a crisis in region 1 and the fundamentals in region 2, which captures wake-up-call contagion.

4 Enhanced perception of risk

To further explore the wake-up-call contagion result, we study the impact of changes in the proportion of informed investors on the probability of a crisis in region 2 when fundamentals are uncorrelated. The proportion of informed investors remains exogenous in this section, but it will be endogenized later. Here we establish an additional result that contributes to the empirical literature on contagion. This literature studies the channels of contagion and the characteristics that make regions susceptible to contagion. We highlight the role of the fundamental in the initially affected region and its non-linear effects on contagion.

Proposition 3 ***Enhanced perception of risk after a wake-up call.*** *Suppose private information is sufficiently precise, public information is sufficiently imprecise, and the prior is strong. After a crisis in region 1 triggered by an intermediate realized fundamental $\theta_1 \in (\underline{\theta}_1, \mu)$, the probability of a crisis in region 2 in case of uncorrelated fundamentals increases in the proportion of informed investors:*

$$\frac{d}{dn}\Big(\Pr\{\theta_2 < \theta_2^*(n,\rho,\theta_1)|\rho=0,\theta_1\}\Big) > 0\,,\ \forall\,\theta_1 \in (\underline{\theta}_1,\mu], \tag{14}$$

where the lower bound $\underline{\theta}_1$ is defined by

$$\underline{\theta}_1 \equiv \mu + \frac{1}{\rho_H}\left(\begin{array}{c}(\theta_2^*(1,0,\theta_1)-\mu)\big[1-\frac{\alpha}{\alpha_2(\rho_H)}\sqrt{\frac{\alpha_2(\rho_H)+\beta}{\alpha+\beta}}\big] \\ +\frac{\sqrt{\beta}}{\alpha_2(\rho_H)}\,\Phi^{-1}(\theta_2^*(1,0,\theta_1))\big[\sqrt{\frac{\alpha_2(\rho_H)+\beta}{\alpha+\beta}}-1\big]\end{array}\right) < \mu. \tag{15}$$

Proof See Appendix B.5. The lower bound $\underline{\theta}_1$ is derived in the proof of Lemma 3.

Figure 1 helps us understand the intuition of Proposition 3. Since fundamentals may be positively correlated, a crisis in region 1 reduces the mean of the updated prior about the fundamental in region 2. Therefore, if more investors are informed, more investors learn that fundamentals are uncorrelated, thus revising upward both the *mean* and the *variance* of the local fundamental θ_2. This *variance effect* enhances *disagreement* among investors, as their posteriors about the local fundamental become more dispersed. For a strong prior, the *mean effect* and the *variance effect* move in opposite directions. Thus, the overall effect of the reassessment of the local fundamental depends on the relative size of both effects.[9]

Mean effect If more investors are informed about the zero correlation of fundamentals, more investors reassess the mean of the local fundamental upward. Better public information – a higher mean of the updated prior $\mu_2(\rho,\theta_1)$ – reduces the fundamental threshold (Vives 2005; Manz 2010). Consequently, $\theta_2^*(1,0,\theta_1)$ is *lower* relative to $\theta_2^*(1,\rho_H,\theta_1)$. This mean effect works against the desired result of enhanced perception of risk after a wake-up call.

[9]See Appendix B.4 for comparative static results and their dependence on these effects. We also provide further intuition on the role of the lower bound $\underline{\theta}_1$ there.

Variance effect If more investors are informed, more investors reassess the precision of the local fundamental downward. More-dispersed public information – a higher variance of the updated prior $\alpha_2(\rho, \theta_1)$ – leads to relatively more-precise private information. This induces greater disagreement among informed investors about the local fundamental. The fundamental threshold increases in the degree of disagreement if the prior about the fundamental is strong (Metz 2002).[10] Investors attack more aggressively, so $\theta_2^*(1,0,\theta_1)$ is *higher* relative to $\theta_2^*(1,\rho_H,\theta_1)$. This variance effect works in favor of the enhanced perception of risk after a wake-up call, provided that the prior is strong.

The probability of a crisis in region 2 increases in the proportion of informed investors if the variance effect dominates the mean effect. Thus, a sizable variance effect is at the heart of the result on the enhanced perception of risk. This label arises since the result is driven by the enhanced disagreement of informed investors and the associated greater concern for the attacking behavior of other investors (strategic uncertainty).[11] The variance effect outweighs the mean effect under the conditions of Lemma 3, namely the lower bound $\underline{\theta}_1$ that restricts the size of the mean effect.

Figure 2 illustrates this link between the fundamental thresholds and the proportion of informed investors. Proposition 3 implies the ranking of fundamental thresholds $\theta_2^*(1,0,\theta_1) > \theta_2^*(0,0,\theta_1)$. For zero realized correlation, there is a one-to-one mapping between the ranking of thresholds and of the probabilities of a crisis. This ranking extends to any proportion of informed investors, $n \in (0,1)$, whereby more-informed investors increase the probability of a crisis in region 2.[12]

Formally, Lemma 4 in Appendix B.3.2 states that the fundamental thresh-

[10]Related to Metz (2002), Heinemann and Illing (2002) show how more transparency reduces speculative attacks, which corresponds to the role of the variance effect discussed in our model. See also Iachan and Nenov (2014) for an investigation of the sensitivity of the net payoffs to the fundamentals when the relative precision of private information changes.

[11]Greater disagreement after a wake-up call is consistent with "an enhanced perception of risk" after the Russian crisis (Van Rijckeghem and Weder 2001, p. 294). Hence, our theory can explain, for example, the unexpected spread of the Russian crisis to Brazil in 1998 (Bordo and Murshid 2000; Forbes 2012) and similar instances during the Asian crisis in 1997 (Radelet and Sachs 1998; Corsetti et al. 1999). See also Pavlova and Rigobon (2008).

[12]This is an uninformed-is-bliss feature. More information can lead to adverse outcomes in Hirshleifer (1971). Information acquisition can be privately optimal but has a negative public value, since it makes co-insurance for risk-averse agents infeasible. Instead, Morris and Shin (2007) analyze optimal communication and provide a rationale for coarse information, for instance in credit ratings. Dang et al. (2012) provide an "ignorance-is-bliss" argument, whereby information insensitivity is key to security design in the money market. More transparency can also be harmful in an expert model with career concerns (Prat 2005).

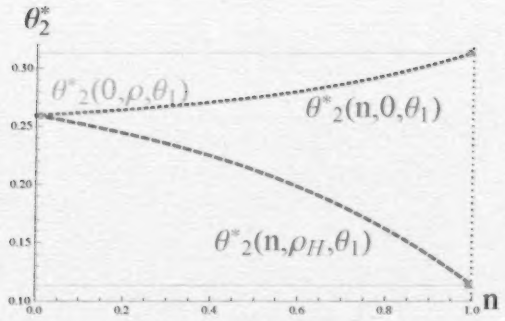

Figure 2: The fundamental thresholds and the proportion of informed investors. Parameters: $\mu = 0.8$, $\alpha = 1$, $\beta = 1$, $b_2 = \ell_2 = 1$, $p = 0.7$, $\rho_H = 0.7$, $\theta_1 = 0.7 < \mu$.

olds evolve continuously and monotonically in the proportion of informed investors, provided sufficiently precise private and sufficiently imprecise public information. In particular, the distance, $|\theta_2^*(n,0,\theta_1) - \theta_2^*(n,\rho_H,\theta_1)|$, continuously increases in the proportion of informed investors, so the fundamental thresholds for $\rho = 0$ and $\rho = \rho_H$ diverge. Intuitively, informed investors capitalize on their information advantage. While uninformed investors must use the same signal threshold irrespective of the realized correlation, informed investors adjust their signal thresholds.[13]

Testable implication There is a large literature on interdependence and contagion in international finance and financial economics with different approaches (see Forbes 2012 for a survey).[14] See Glick and Rose (1999), Van Rijckeghem and Weder (2001, 2003), and Dasgupta et al. (2011) for an empirical literature investigating (i) the channels of contagion during financial crises; and (ii) the dependence on the characteristics of the affected countries. This literature suggests that stronger trade or financial links and higher institutional similarity increase contagion.

In our model, the correlation of regional fundamentals captures such factors: $\rho \geq 0$ measures the intensity of trade or financial links and institutional similarities with the initially affected region, which has fundamentals θ_1. Let $\Pr(\theta_2, \rho)$ be

[13] A larger proportion of informed investors raises the fundamental threshold $\theta_2^*(n,0,\theta_1)$, as investors attack more aggressively after learning $\rho = 0$, compared with uninformed investors (Part (a) of Lemma 4; see dotted line in Figure 2). The opposite holds for a positive correlation, $\rho = \rho_H$, when informed investors attack relatively less aggressively, so $\theta_2^*(n,\rho_H,\theta_1)$ decreases in the proportion of informed investors (dashed line in Figure 2). Finally, the difference between these thresholds increases in the proportion of informed investors (Part (b) of Lemma 4).

[14] The approaches include probability models (Eichengreen et al. 1996), correlation analysis (Forbes and Rigobon 2002), VAR models (Favero and Giavazzi 2002), latent factor/GARCH models (Bekaert et al. 2014), and extreme value analysis (Bae et al. 2003).

the probability of a crisis in another region with the characteristics θ_2 and ρ. This probability is conditional on a crisis in region 1. Consistent with the empirical literature, our model (and the related theoretical literature on information contagion) predicts that $d\left(\frac{d\Pr(\theta_2,\rho)}{d\rho}\right)/d\theta_1 < 0$. More importantly, our model also predicts a non-linearity in θ_1 due to the enhanced perception of risk after a wake-up call.

Empirical prediction

$$\begin{aligned} \frac{d\Pr(\theta_2,\rho)}{d\rho} &< 0 \quad if \quad \theta_1 > \underline{\theta}_1 \\ \frac{d\Pr(\theta_2,\rho)}{d\rho} &> 0 \quad if \quad \theta_1 < \underline{\theta}_1. \end{aligned} \tag{16}$$

This prediction is based on the variance effect (see Lemma 3 and Proposition 3). In particular, after controlling for the contemporaneous fundamentals of the second region, θ_2, there is a non-linear effect of the realized fundamental in the first region, θ_1. A crisis in the first region due to *moderately low* fundamentals is *more* likely to spread if the empiricist observes no linkages, $\rho = 0$. By contrast, a crisis due to *extremely low* fundamentals is *less* likely to spread if the empiricist observes no linkages, which is consistent with existing empirical findings.

In sum, our wake-up-call theory of contagion suggests a role for the fundamentals of the initially affected region. Furthermore, these fundamentals also drive the direction of the effect of an increase in ρ. Therefore, an empiricist should discriminate between moderately low and extremely low realizations of fundamentals in the initially affected region. Our theory suggest that this may improve the measurement of contagion, especially for currency attacks and bank runs.

5 Endogenous information

We discuss the value of information in section 5.1. We show that the private value of information about the correlation increases in the proportion of informed investors. Next, we study the costly acquisition of information about this correlation in section 5.2, where we describe conditions sufficient for the existence of a unique equilibrium with wake-up-call contagion. This result hinges on the second key feature of our model (informational asymmetry), which makes information more easily avail-

able to investors in region 2 after a crisis in region 1. In section 5.3, we discuss how this assumption can be relaxed to obtain that information acquisition occurs only after a crisis in region 1, despite symmetric availability of information (or cost of information). Finally, we show in section 5.4 that our key results are enhanced when allowing for endogenous precision of private information.

5.1 The value of information

Information about the realization of the correlation of fundamentals has value to an individual investor. Its value is the difference between the expected utility term of informed investors, EU_I, and of uninformed investors, EU_U. These expected utilities are defined in Appendix B.6. The expected utility of an informed investor takes into account the possible realizations of the correlation, since these affect the signal threshold of an informed investor, $x_I^*(n,0,\theta_1)$ and $x_I^*(n,\rho_H,\theta_1)$. By contrast, an uninformed investor cannot tailor the attack strategy and must use the *same* signal threshold $x_U^*(n,\theta_1)$ throughout.

Let $v(n,\theta_1) \equiv EU_I - EU_U$ be an individual investor's value of information about ρ conditional on θ_1 and the proportion of informed investors n:

$$v(n,\theta_1) = p\left(\begin{array}{l} \int_{-\infty}^{\theta_2^*(n,0,\theta_1)} b_2 \int_{x_U^*(n,\theta_1)}^{x_I^*(n,0,\theta_1)} g(x_{i2}|\theta_2)dx_{i2} f(\theta_2|0,\theta_1)d\theta_2 \\ -\int_{\theta_2^*(n,0,\theta_1)}^{+\infty} \ell_2 \int_{x_U^*(n,\theta_1)}^{x_I^*(n,0,\theta_1)} g(x_{i2}|\theta_2)dx_{i2} f(\theta_2|0,\theta_1)d\theta_2 \end{array}\right) - \quad (17)$$
$$(1-p)\left(\begin{array}{l} \int_{-\infty}^{\theta_2^*(n,\rho_H,\theta_1)} b_2 \int_{x_I^*(n,\rho_H,\theta_1)}^{x_U^*(n,\theta_1)} g(x_{i2}|\theta_2)dx_{i2} f(\theta_2|\rho_H,\theta_1)d\theta_2 \\ -\int_{\theta_2^*(n,\rho_H,\theta_1)}^{+\infty} \ell_2 \int_{x_I^*(n,\rho_H,\theta_1)}^{x_U^*(n,\theta_1)} g(x_{i2}|\theta_2)dx_{i2} f(\theta_2|\rho_H,\theta_1)d\theta_2 \end{array}\right),$$

where the distribution of the fundamental conditional on the realized correlation, $f(\theta_2|\rho,\theta_1)$, is normal with mean $\mu_2(\rho,\theta_1)$ and precision $\alpha_2(\rho)$, and the distribution of the private signal conditional on the fundamental, $g(x|\theta_2)$, is normal with mean θ_2 and precision β. In Appendix B.6, we provide intuition for the benefits of a tailored signal threshold used by informed investors. We also describe the type-I and type-II errors that investors make in their attack behavior.

Proposition 4 states how the value of information changes with the proportion of informed investors. Hellwig and Veldkamp (2009) show that information choices

inherit the strategic complementarity or substitutability from the underlying beauty contest game.[15] We show that this inheritance result extends to a global coordination game of regime change, particularly in the context of ex-ante uncertainty about the correlation of fundamentals and where information about the correlation becomes publicly available to a fraction n of informed investors.

Proposition 4 ***Strategic complementarity in information choices.*** *Suppose the prior about fundamentals in region* 2 *is strong, private information is precise,* $\beta > \underline{\beta}_2 < \infty$, *and public information is imprecise,* $0 < \alpha < \overline{\alpha}$. *After a crisis in region* 1, *the value of information increases in the proportion of informed investors:*

$$\frac{dv(n,\theta_1)}{dn} \geq 0 \ \forall \ \theta_1 < \mu. \tag{18}$$

Furthermore, for any proportion of informed investors, $n \in [0,1]$, *we have:*

$$v(n,\underline{\theta}_1) = 0; \quad v(n,\theta_1) > 0 \ \forall \ \theta_1 \neq \underline{\theta}_1. \tag{19}$$

Proof See Appendix B.7.

If $\theta_1 = \underline{\theta}_1$, then the signal thresholds of informed and uninformed investors coincide, $x_I^* = x_U^*$. In this special case, both the *private* and the *social value* of information about ρ is zero, since the attacking strategies do not depend on the additional information. For the general case of $\theta_1 \neq \underline{\theta}_1$, a strategic complementarity arises because individual investors benefit from knowing what others know, as in Hellwig and Veldkamp (2009). Formally, the signal thresholds $x_I^*(n,0,\theta_1)$ and $x_I^*(n,\rho_H,\theta_1)$ diverge when n increases. As a result, it is more likely that the individual attack decision of an informed investor is adjusted the more others are informed. Hence, the resulting private value of information increases. This property arises from the monotonicity in signal thresholds (see Lemma 4 C in Appendix B.3.2).

In line with existing literature, the private value of information about ρ is *always* non-negative, while the social value of information about ρ may be *positive*

[15] Ahnert and Kakhbod (2014) obtain strategic complementarity in information choices in a one-region global coordination game of regime change with a common prior, a discrete private information choice and heterogeneous information costs. They show that the information choice of investors amplifies the probability of a financial crisis.

or *negative*. As demonstrated by Morris and Shin (2002) in a beauty contest game, a change in the precision of public information can have an ambiguous effect on speculators' welfare if publicity is high (Cornand and Heinemann 2008). In our global games model, there is an additional layer due to the variation of the equilibrium fundamental thresholds in n (see Figure 2), together with the interplay of the mean effect and the variance effect. If the information about ρ, which is publicly available to informed investors, is unfavorable for the fundamentals of region 2 or causes a decrease in disagreement, then the social value of information about ρ (from the viewpoint of investors) is positive. This is because both the likelihood of a crisis and the expected payoff from attacking increase. Instead, information about ρ that increases disagreement among informed investors may have a negative social value, as in Morris and Shin (2002).

5.2 Endogenous information about ρ

In this section, we endogenize the information investors use to reassess the local fundamental after a wake-up call. We study the costly acquisition of information about ρ, which helps to improve the forecast about θ_2. Therefore, the information stage at $t = 2$ is modified by introducing a simultaneous information choice game, where investors decide after a crisis whether to purchase a perfectly revealing and publicly available signal about ρ at a cost $c > 0$. Each investor can purchase the same signal and observes it privately.[16] After a crisis elsewhere, more information is produced due to news coverage and public inquiries. Hence, we assume that information is more easily available after a crisis in region 1 (the second key feature of our model), thereby imposing an informational asymmetry that will be relaxed in section 5.3.

We analyze pure-strategy perfect Bayesian equilibrium (PBE) in threshold strategies (Definition 2). Let $d_i \in \{I, U\}$ denote the information choice of investor i and let $a_{iI} \equiv a_{i2}(d_i = I)$ and $a_{iU} \equiv a_{i2}(d_i = U)$ denote the corresponding attack rules.[17] We show that the fundamental reassessment after a wake-up call – the heart

[16] In terms of wholesale investors or currency speculators, costly information acquisition could be access to Bloomberg and Datastream terminals, or the hiring of analysts who assess the publicly available data. Our results are also robust to the introduction of noisy signals about the correlation.

[17] In contrast to section 4, we no longer need to assume common knowledge about the proportion of informed investors. Furthermore, under the stated conditions on the information cost, the

of our contagion mechanism – arises endogenously in the unique equilibrium when the information cost is sufficiently low after a crisis (Proposition 5).

Definition 2 *A pure-strategy monotone perfect Bayesian equilibrium comprises an information choice $d_i^* \in \{I, U\}$ for each investor $i \in [0,1]$, an aggregate proportion of informed investors $n^* \in [0,1]$, an attack rule $a_{i2d}^*(n^*; \theta_1, x_{i2}) \in \{0,1\}$ for each investor, and an aggregate attack size $A_2^* \in [0,1]$ such that:*

1. *All investors optimally choose d_i at the information stage.*
2. *The proportion n^* is consistent with the individually optimal information choices $\{d_i^*\}_{i\in[0,1]}$.*
3. *Uninformed investors have an optimal attack rule $a_{2U}^*(n^*; \theta_1, x_{i2})$. For any given realization of $\rho \in \{0, \rho_H\}$, informed investors have an optimal attack rule $a_{2I}^*(n^*; \theta_1, \rho, x_{i2})$.*
4. *The proportion A_2^* is consistent with the individually optimal attack decisions:*

$$\begin{aligned} A_2^* \equiv A(n^*; \theta_2, \rho) = n^* \int_{-\infty}^{+\infty} a_{2I}^*(n^*; \theta_1, \rho, x_{i2}) \sqrt{\beta}\phi(\sqrt{\beta}(x_{i2} - \theta_2))dx_{i2} \quad (20) \\ + \quad (1-n^*) \int_{-\infty}^{+\infty} a_{2U}^*(n^*; \theta_1, x_{i2}) \sqrt{\beta}\phi(\sqrt{\beta}(x_{i2} - \theta_2))dx_{i2}, \quad \forall \rho \in \{0, \rho_H\}. \end{aligned}$$

Proposition 5 ***Existence of a unique equilibrium with wake-up-call contagion.*** *Suppose the prior about the fundamentals in region 2 is strong, private information is precise, $\beta > \max\{\underline{\beta}_2, \underline{\beta}_4\} < \infty$, and public information is imprecise, $\alpha < \overline{\alpha} > 0$. After a crisis in region 1, there exists a unique monotone pure-strategy PBE if the information cost is sufficiently small, $c < \overline{c}(0, \theta_1)$. All investors acquire information, $n^* = 1$, and use the signal threshold $x_I^*(1, \rho, \theta_1)$ for each $\rho \in \{0, \rho_H\}$. Even if fundamentals are uncorrelated, contagion occurs after a wake-up call.*

Proof See Appendix B.8.

Proposition 5 states that the fundamental reassessment after a wake-up call entails the acquisition of information about the correlation of fundamentals in dominant actions, $n^* = 1$, for a small positive cost $c \in (0, \overline{c}(0, \theta_1))$, for any $\theta_1 \neq \underline{\theta}_1$.

information choice of investors is in dominant actions. Based on the previous analysis, such as Proposition 1 and Lemma 4, we study the optimal information choice d_i^*. Proposition 4 lays the foundation for a strategic complementarity in information choices.

Thus, contagion after a wake-up call arises endogenously – even if investors learn that fundamentals are uncorrelated.

5.3 Relaxing our assumption on informational asymmetry

The result of wake-up-call contagion, namely the comparison in equation (13), requires (i) learning about uncorrelated fundamentals after a crisis in region 1 and (ii) no learning about the correlation after no crisis in region 1. These requirements are assured by the second key feature of our model – the informational asymmetry – that makes information more easily (or cheaply) available to investors in region 2 after a crisis in region 1. In this section, we discuss a relaxation of the information asymmetry assumption that preserves our key insight on wake-up-call contagion.

One way to fully relax the assumption about the informational asymmetry is to introduce an aggregate macro shock that may hit both regions simultaneously. A negatively skewed macro shock can create genuinely higher incentives to become informed about the correlation after a crisis in region 1, compared to no crisis. Hence, information acquisition does not take place after no crisis in 1 – even if the information cost is independent of the realization of θ_1. (Recall that we have assumed so far that information acquisition is too costly after no crisis in region 1.)

To see this, recall from section 5.1 that the benefit of information about the correlation increases in the difference in equilibrium signal thresholds, or equivalently, in the difference in fundamental thresholds. The existence of a negatively skewed macro shock results in a higher weight on those states of the world in which the fundamental thresholds differ whenever a crisis was observed in region 1. As a result, the incentives to acquire information about the correlation (that is, the exposure to the macro shock) are higher after observing a crisis elsewhere.

To illustrate the mechanics, consider a simplified version of our model, where direct fundamental links are absent, $\rho_H = 0$. Suppose that, with probability p, both regions are simultaneously exposed to a macro or common shock that is the only potential link between the two regions. Thus, both regions are not exposed with probability $1 - p$. The random macro shock, m, may be positive or negative and it affects the fundamental of each exposed region additively:

$$\hat{\theta}_i = m + \theta_i, \quad \theta_i \sim \mathcal{N}(\mu, \alpha^{-1}). \tag{21}$$

We assume that the macro shock takes on a small positive value $m = \Delta > 0$ with probability $1-q$, and a large negative value $m = -s\Delta < 0$ with probability q, where $s > 1$ is a scaling factor. We impose $q(1-s) = 1$ to ensure a zero mean, $E[m] = 0$.

If the exposure to the macro shock and its realization are common knowledge, then $(\theta_i^* | exposure, m < 0) > (\theta_i^* | no\ exposure) > (\theta_i^* | exposure, m > 0)$. Instead, if the macro shock is unobserved, then learning about a crisis (or about no crisis) in region 1 leads to Bayesian updating. Both the conditional probabilities about the exposure to the macro shock and its sign are updated. Observing a crisis leads to an increase (decrease) in the conditional probability of being exposed to a negative (positive) macro shock. Furthermore, the conditional probability of not being exposed to a macro shock also decreases. The opposite updating takes place after observing no crisis in 1. Crucially, the incentives to acquire information about the exposure to the macro shock are higher after observing a crisis, since the benefits of a tailored signal threshold increase when a more-extreme state is more likely.

5.4 Endogenous precision of private information

In section 5.2, we analyzed endogenous information about the correlation of fundamentals, which helps investors in region 2 reassess the local fundamental θ_2. In this section, we extend our analysis to the private information choice about the local fundamental θ_2.[18] In particular, we consider a model where investors choose the precision of their private information subject to convex information costs, as in Szkup and Trevino (2012). In this set-up, the acquired information about θ_2 is by definition not correlated, whereas the acquired information about ρ is correlated. In sum, we show that our result of wake-up-call contagion is further strengthened under private information choice.

After observing a crisis in region 1, investors in region 2 simultaneously choose the precision of their signal about θ_2. To simplify the exposition, we restrict attention to the case when the information cost for the signal about the correlation is sufficiently low, such that all investors learn the realized correlation ρ after

[18] We thank our discussant Laura Veldkamp for suggesting that we analyze this case.

the wake-up call. Szkup and Trevino (2012) develop a single-region global games model with a related payoff structure:

$$\begin{aligned} u(a_i = 1, A, \theta) &= (1-T)\ 1_{\{A > 1-\theta\}} - T\ 1_{\{A \leq 1-\theta\}} \\ u(a_i = 0, A, \theta) &= 0, \end{aligned} \tag{22}$$

where $\theta \sim \mathcal{N}\left(\mu_\theta, \tau_\theta^{-1}\right)$ is unobserved but investors receive the private signal $x_i|\theta \sim \mathcal{N}\left(\theta, \tau^{-1}\right)$. For the special case of $T = 1/2$ and $b_2 = \ell_2 = 1/2$, we have an equivalent formulation, where we just insert the subscript for region 2:

$$\begin{aligned} u(a_{i2} = 1, A_2, \theta_2) &= 1/2\ 1_{\{A_2 > 1-\theta = \theta_2\}} - 1/2\ 1_{\{A_2 \leq 1-\theta = \theta_2\}} \\ u(a_{it} = 0, A_2, \theta_2) &= 0, \end{aligned} \tag{23}$$

where $\theta_2 \sim \mathcal{N}\left(\mu_2, \alpha_2^{-1}\right)$, with $\mu_2 = 1 - \mu_\theta$ and $\alpha_2 = \tau_\theta$.

Szkup and Trevino (2012) show that there exists a unique equilibrium in the information game under certain assumptions on the convex cost function for acquiring more-precise private signals. In Appendix B.9 of Ahnert and Bertsch (2015), we specify these assumptions and derive the benefit of a higher private signal precision for investors in region 2, where investors learn about the correlation after a crisis in region 1. We show that this benefit function is identical to the one derived by Szkup and Trevino.

Furthermore, building on the results of Szkup and Trevino (2012), we find that the marginal benefit of increasing the precision of private information decreases in the precision of public information, provided the prior is sufficiently strong. Extending their analysis, we show that the marginal benefit of increasing the private signal precision decreases in the mean of public information if the prior about the fundamental in region 2 is sufficiently strong.

Formally, for the special case of $b_2 = \ell_2 = 1/2$, we find that a decrease in α_2 has two effects on the fundamental threshold. Both effects go in the same direction and increase θ_2^* (as well as the probability of a crisis in region 2). First, $d\theta_2^*/d\alpha_2 < 0$ for a given level of β_2 and, second, $d\beta_2^*/d\alpha_2 < 0$, which also decreases θ_2^* because $d\theta_2^*/d\beta_2 > 0$. Furthermore, we find that an increase in μ_2 also has two effects that go in the same direction and both decrease θ_2^*. First, $d\theta_2^*/d\mu_2 < 0$ and, second, $d\beta_2^*/d\mu_2 < 0$, which also decreases θ_2^* because $d\theta_2^*/d\beta_2 > 0$.

Taken together, these results imply that the wake-up-call contagion result of Proposition 2 can be strengthened if the prior is sufficiently strong. The strengthening of the result is reflected in the endogenous private signal precisions, which further increase the difference in the equilibrium fundamental thresholds, θ_2^*:

$$\Pr\{\theta_2 < \theta_2^*(n=1,\rho,\theta_1;\beta_2^*)|\rho=0,\theta_1<\theta_1^*\} > \Pr\{\theta_2 < \theta_2^*(n=0,\rho,\theta_1;\beta_2^*)|\theta_1 \geq \theta_1^*\}, \quad (24)$$

where the optimal precision of private information after a wake-up call and learning that fundamentals are uncorrelated is higher than after no wake-up call:

$$[\beta_2^*|\rho=0,\theta_1<\theta_1^*] > [\beta_2^*|\theta_1 \geq \theta_1^*]. \quad (25)$$

Intuitively, the private signal precision is relatively higher on the left-hand side for two reasons. First, the zero correlation makes public information more dispersed (decrease in α_2), which leads to a relatively higher θ_2^* on the left-hand side. Second, not observing a crisis in region 1 means that the fundamental in region 1 must have been good. This leads to an upward revision in μ_2 and, hence, to a decrease in the optimally chosen precision of private information. This effect is associated with a relatively lower θ_2^* on the right-hand side.

6 Conclusion

We propose a theory of financial contagion that explains how wake-up calls transmit crises. We study global coordination games of regime change in two regions with ex-ante uncertainty about the correlation of fundamentals. A crisis in region 1 is a wake-up call for investors in region 2 that induces a reassessment of the local fundamental and an increase in the probability of a crisis in region 2. Contagion occurs even in the absence of ex-post correlated fundamentals, common lenders and balance-sheet links. Thus, we isolate the wake-up-call component of contagion.

There are two reasons for contagion to arise even if investors learn that fundamentals in region 2 are uncorrelated with those in region 1. First, the mean of the fundamental in region 2 is lower after the wake-up call. Not observing a crisis

in region 1 would have been favorable news for fundamentals in region 2, since the correlation of fundamentals may be positive. This mean effect increases the probability of a crisis in region 2. Second, the variance of the fundamental in region 2 is higher after the wake-up call. When fundamentals are uncorrelated, observing a crisis in region 1 is uninformative for investors in region 2. Hence, there is greater disagreement among informed investors. This variance effect can increase the probability of a crisis in region 2. Both effects are aligned and induce investors to attack the regime more aggressively, leading to contagion after a wake-up call.

We derive these results under the condition that information is more easily available after a crisis. We argue that our results prevail when this informational asymmetry is relaxed. The results are also robust to the introduction of private information choice, which further enhances the disagreement effect. The result on wake-up-call contagion (Proposition 2) is also robust to introducing imperfect information about both the correlation and region 1's fundamental after a crisis.

The wake-up-call theory of contagion has several applications. Currency speculators observe an exchange rate crisis elsewhere and are uncertain about the magnitude of trade and financial links. Uninsured bank creditors observe a run elsewhere and are uncertain about interbank linkages. Sovereign debt holders observe a default elsewhere and are uncertain about the resources and commitment of multilateral bailout funds or the international lender of last resort.

Our theory of contagion is consistent with existing evidence and creates a new testable implication. We derive the empirical prediction that contagion depends non-linearly on the fundamental in the region of the initial crisis. Our implications are also attractive for experimental work, where the information choice is observed.

We wish to study implications for welfare and policy in subsequent work.

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A Equilibrium

A.1 Deriving the equilibrium in region 2 after a crisis in 1

We proceed in two steps. First, we consider the special case in which all investors are informed ($n = 1$) in section A.1.1. The existence of a unique Bayesian equilibrium is just a corollary of Result 1 in this case. Second, we derive the equilibrium conditions for the general case in which some investors are uninformed ($n \in (0,1]$) in section A.1.2. Third, we extend the existence and uniqueness result to the special case where all investors are uninformed ($n = 0$) in section A.1.3. The results are summarized in Lemma 2. In Appendix A.2, we subsequently prove the existence of a unique monotone equilibrium for the general case $n \in (0,1]$. We also extend the result of Lemma 2 to the case of no crisis in region 1.

A.1.1 All investors are informed

When all investors are informed, $n = 1$, they learn the realized correlation. In the case of a zero correlation, the updated prior of informed investors in region 2 coincides with that of investors in region 1 and the previous analysis applies. In the case of a positive correlation, by contrast, a small change is required to obtain a corollary of Result 1. The modified threshold for the precision of private information is $\underline{\beta}_0' \equiv \frac{\alpha^2}{2\pi(1-\rho_H^2)^2} \in (\underline{\beta}_0, \infty)$. Moreover, the unique threshold fundamental $\theta_2^* = \theta_2^*(n = 1, \rho, \theta_1)$ is implicitly defined by

$$I_2^*(\theta_2^*, \rho) \equiv \Phi\left(\frac{\alpha_2(\rho)\left[\theta_2^* - \mu_2(\rho, \theta_1)\right]}{\sqrt{\alpha_2(\rho)+\beta}} - \sqrt{\frac{\beta}{\alpha_2(\rho)+\beta}}\Phi^{-1}(\theta_2^*) \right) = \gamma_2, \qquad (26)$$

for any realized correlation $\rho \in \{0, \rho_H\}$ and any observed fundamental $\theta_1 < \theta_1^*$.

Corollary 1 *Suppose all investors are informed about the correlation, $n = 1$, after a crisis in region 1, $\theta_1 < \theta_1^*$. If private information is sufficiently precise, $\beta > \underline{\beta}_0'$, then there exists a unique Bayesian equilibrium in region 2. This equilibrium is in threshold strategies, whereby a crisis occurs if the realized fundamental is below a threshold $\theta_2^*(1, \rho, \theta_1)$ defined by equation (26).*

A.1.2 Some investors are uninformed

Consider next the general case of $n \in [0,1)$. After a crisis in region 1, uninformed investors use the observed θ_1 and their private signal x_{i2} to reassess the local fundamental θ_2. Uninformed investors do not learn the correlation of fundamentals.

Bayesian updating We show that the relationship between the posterior probability of a zero correlation, $\hat{p}$, and the private signal, x_{i2}, is non-monotone. First, $\frac{d\hat{p}}{dx_{i2}} > 0$ if the private signal is relatively high. Intuitively, an investor places more weight on the probability of a zero correlation after receiving a relatively good private signal. Instead, after a low private signal, $\frac{d\hat{p}}{dx_{i2}} > 0$ is not guaranteed. For extremely low signals, an even worse signal makes an uninformed investor infer that $\rho = 0$ is more likely due to the fatter tails of the more-dispersed prior. Uninformed investors use Bayes' rule to form a belief about the correlation of fundamentals:

$$\hat{p} \equiv \Pr\{\rho = 0 | \theta_1, x_{i2}\} = \frac{p \Pr\{x_{i2}|\theta_1, \rho = 0\}}{p \Pr\{x_{i2}|\theta_1, \rho = 0\} + (1-p)\Pr\{x_{i2}|\theta_1, \rho = \rho_{II}\}}. \quad (27)$$

Computing $\Pr\{x_{i2}|\theta_1, \rho\}$ for each ρ, recall that the variance is independent of θ_1:

$$\begin{aligned} \Pr\{x_{i2}|\theta_1, \rho = 0\} &= \frac{1}{\sqrt{\mathrm{Var}[x_{i2}|\rho = 0]}} \phi\left(\frac{x_{i2} - \mathbb{E}[x_{i2}|\theta_1, \rho = 0]}{\sqrt{\mathrm{Var}[x_{i2}|\rho = 0]}}\right) \\ &= \left(\frac{1}{\alpha} + \frac{1}{\beta}\right)^{-\frac{1}{2}} \phi\left(\frac{x_{i2} - \mu}{\sqrt{\frac{1}{\alpha} + \frac{1}{\beta}}}\right) \quad (28) \\ \Pr\{x_{i2}|\theta_1, \rho = \rho_{II}\} &= \frac{1}{\sqrt{\mathrm{Var}[x_{i2}|\rho = \rho_{II}]}} \phi\left(\frac{x_{i2} - \mathbb{E}[x_{i2}|\theta_1, \rho = \rho_{II}]}{\sqrt{\mathrm{Var}[x_{i2}|\rho = \rho_{II}]}}\right) \\ &= \left(\frac{1-\rho_{II}^2}{\alpha} + \frac{1}{\beta}\right)^{-\frac{1}{2}} \phi\left(\frac{x_{i2} - [\rho_{II}\theta_1 + (1-\rho_{II})\mu]}{\sqrt{\frac{1-\rho_{II}^2}{\alpha} + \frac{1}{\beta}}}\right) \quad (29) \end{aligned}$$

Since $\rho_{II} > 0$, the derivatives of the posterior belief $\hat{p}$ are

$$\frac{d\hat{p}}{d\theta_1} \begin{cases} \geq 0 & \text{if } x_{i2} \leq \rho_{II}\theta_1 + (1-\rho_{II})\mu \\ < 0 & \text{otherwise.} \end{cases} \quad (30)$$

First, if the private signal x_{i2} is sufficiently low, an increase in θ_1 induces uninformed investors to put a larger probability on uncorrelated regional fundamentals.

The signs of this derivative would be reversed if we had $\rho_H < 0$.

Second, how does $\hat{p}$ vary with the private signal x_{i2}? We find that

$$\frac{d\hat{p}}{dx_{i2}} \begin{cases} > & 0 \text{ if } \rho_H > 0 \text{ and } x_{i2} \geq \rho_H \theta_1 + (1-\rho_H)\mu \\ < & 0 \text{ if } \rho_H < 0 \text{ and } x_{i2} \leq \rho_H \theta_1 + (1-\rho_H)\mu \\ \lesseqgtr & 0 \text{ otherwise.} \end{cases} \tag{31}$$

Therefore, after receiving a relatively good private signal, $x_{i2} \geq \rho_H \theta_1 + (1-\rho_H)\mu$, an investor places more weight on the probability of zero cross-regional correlation. If the private signal takes an intermediate value, $\frac{d\hat{p}}{dx_{i2}} > 0$ still holds. However, after receiving a relatively low private signal, $x_{i2} < \rho_H \theta_1 + (1-\rho_H)\mu$, we have that $\frac{d\hat{p}}{dx_{i2}} \leq 0$ due to the more-dispersed prior distribution if $\rho = 0$. For the same reason, an extremely high or low private signal induces uninformed investors to believe that fundamentals are uncorrelated across regions, $\lim_{x_{i2} \to +\infty} \hat{p} = 1 = \lim_{x_{i2} \to -\infty} \hat{p}$.

Equilibrium conditions when some investors are uninformed Analyzing the general case of some uninformed investors, we derive the system of equations – the critical mass and indifference conditions – describing the equilibrium in region 2.

The critical mass conditions state that the proportion of attacking investors $A_2^*(\rho)$ equals the fundamental threshold $\theta_2^*(\rho)$ for each realized $\rho \in \{0, \rho_H\}$:

$$\theta_2^*(\rho) = n\Phi\left(\sqrt{\beta}[x_I^*(\rho) - \theta_2^*(\rho)]\right) + (1-n)\Phi\left(\sqrt{\beta}[x_U^* - \theta_2^*(\rho)]\right). \tag{32}$$

We use the short-hands $\theta_2^*(\rho) \equiv \theta_2^*(n, \rho, \theta_1)$, $x_I^*(\rho) \equiv x_I^*(n, \rho, \theta_1)$, and $x_U^* \equiv x_U^*(n, \theta_1)$ for the fundamental threshold and the signal thresholds of informed and uninformed investors, respectively.

The first indifference condition states that an uninformed investor with threshold signal $x_{i2} = x_U^*$ is indifferent between attacking and not attacking:

$$\underbrace{\hat{p}^* \Psi(\theta_2^*(0), x_U^*, 0) + (1-\hat{p}^*)\Psi(\theta_2^*(\rho_H), x_U^*, \rho_H)}_{\equiv J(n, \theta_2^*(0), \theta_2^*(\rho_H), x_U^*)} = \gamma_2, \tag{33}$$

where $\hat{p}^* = \hat{p}(\theta_1, x_U^*)$ and, for $d \in \{I, U\}$ and $\rho \in \{0, \rho_H\}$:

$$\Psi(\theta_{2d}^*, x_d^*, \rho) \equiv \Phi\left(\theta_{2d}^* \sqrt{\alpha_2(\rho) + \beta} - \frac{\alpha_2(\rho)\mu_2(\rho, \theta_1) + \beta x_d^*}{\sqrt{\alpha_2(\rho) + \beta}}\right). \tag{34}$$

Two additional indifference conditions, one for each realized correlation, state that an informed investor is indifferent as to whether to attack upon receiving the threshold signal $x_{i2} = x_I^*(\rho)$:

$$\Psi(\theta_2^*(\rho), x_I^*(\rho), \rho) = \gamma_2 \ \forall \rho \in \{0, \rho_{II}\}. \tag{35}$$

We have five equations in five unknowns. In the simplest case, in region 1, we had two thresholds x_1^* and θ_1^*. There, the objective was to establish aggregate behavior by inserting the critical mass condition, which states x_1^* in terms of θ_1^*, into the indifference condition. This yields one equation implicit in θ_1^*. We pursue a modified strategy here, solving this system of equations in order to express the equilibrium in terms of $\theta_2^*(0)$ and $\theta_2^*(\rho_{II})$ only.

We also use the following insight. Since uninformed investors do not observe the realized cross-regional correlation, the signal threshold must be identical across these realizations, $x_U^*(\rho = 0) = x_U^*(\rho = \rho_{II})$. In the following steps, we derive this threshold for either realization of the correlation ρ by using the fundamental threshold $\theta_2^*(\rho)$, and equalize both expressions. First, we use the critical mass condition in equation (32) for $\theta_2^*(0)$ to express x_U^* as a function of $\theta_2^*(0)$ and $x_I^*(0)$. Second, we use the indifference condition of informed investors in case of $\rho = 0$, equation (35), to obtain $x_I^*(0)$ as a function of $\theta_2^*(0)$. Third, we use the critical mass condition in equation (32) for $\theta_2^*(\rho_{II})$ to express x_U^* as a function of $\theta_2^*(\rho_{II})$ and $x_I^*(\rho_{II})$. Then, we use the indifference condition of informed investors in case of $\rho = \rho_{II}$, equation (35), to obtain $x_I^*(\rho_{II})$ as a function of $\theta_2^*(\rho_{II})$. Thus, $\forall \rho$:

$$x_U^*(\rho) = \theta_2^*(\rho) + \frac{\Phi^{-1}\left(\frac{\theta_2^*(\rho) - \pi\Phi\left(\frac{\alpha_2(\rho)(\theta_2^*(\rho) - \mu_2(\rho,\theta_1)) - \sqrt{\alpha_2(\rho)+\beta}\,\Phi^{-1}(\gamma_2)}{\sqrt{\beta}}\right)}{1-\pi}\right)}{\sqrt{\beta}}. \tag{36}$$

Hence, for $\rho \in \{0, \rho_{II}\}$, a sufficient condition for the partial derivatives with respect to the fundamental thresholds to be strictly positive is $\beta > \underline{\beta}_1$:

$$\frac{dx_U^*(\rho)}{d\theta_2^*(\rho)} > 0. \tag{37}$$

Since the signal threshold is the same for an uninformed investor, subtracting equation (36) evaluated at $\rho = 0$ from the same equation evaluated at $\rho = \rho_{II}$ must

yield zero. This yields the first implicit relationships between $\theta_2^*(0)$ and $\theta_2^*(\rho_H)$:

$$K(n, \theta_2^*(0), \theta_2^*(\rho_H)) \equiv x_U^*(0) - x_U^*(\rho_H) = 0. \tag{38}$$

Next, we construct the second implicit relationship between the two aggregate thresholds $\theta_2^*(0)$ and $\theta_2^*(\rho_H)$ in two steps. First, insert equation (36) evaluated at $\rho = 0$ in $\Psi(\theta_2^*(0), x_U^*(0), 0)$ and in $\hat{p}$ as used in $J(n, \theta_2^*(0), \theta_2^*(\rho_H), x_U^*)$. Second, insert equation (36) evaluated at $\rho = \rho_H$ in $\Psi(\theta_2^*(\rho_H), x_U^*(\rho_H), \rho_H)$. Combining both expressions yields

$$L(n, \theta_2^*(0), \theta_2^*(\rho_H)) \equiv J(n, \theta_2^*(0), \theta_2^*(\rho_H), x_U^*(0), x_U^*(\rho_H)) = \gamma_2. \tag{39}$$

A.1.3 All investors are uninformed

If all investors are uninformed, $n = 0$, the system of equations derived in Appendix A.1.2 simplifies. Specifically, there is only one fundamental threshold and the system can be reduced to one equation in one unknown, where $\theta_2^*(0, 0, \theta_1) = \theta_2^*(0, \rho_H, \theta_1)$ in equation (33).

Using the results of Milgrom (1981) and Vives (2005), we show that the best-response function of an individual investor strictly increases in the threshold used by other investors. Therefore, there exists a unique equilibrium in threshold strategies if private information is sufficiently precise, as proven in the next paragraph.

Monotonicity In contrast to the standard analysis of region 1, $J(0, \theta_2, \theta_1)$ is harder to characterize. The weights of the mixture distribution and the posterior beliefs about the correlation now depend on the threshold signal x_U^*. Therefore, the question arises as to whether our focus on monotone equilibria is justified, in light of the global non-monotonicity of $\hat{p}(x_U^*(\theta_2^*(0, 0, \theta_1)))$ in x_U^* and, hence, in $\theta_2^*(0, 0, \theta_1)$, as established above. Fortunately, the best-response function of an individual investor i is proven to be strictly increasing in the threshold used by other investors:

$$r' = -\frac{\frac{d\Pr\{\theta_2 < \hat{\theta}_2(\hat{x}_2) | \theta_1, x_{i2}\}}{d\hat{x}_2}}{\frac{d\Pr\{\theta_2 < \hat{\theta}_2(\hat{x}_2) | \theta_1, \tilde{x}_{i2}\}}{d\tilde{x}_{i2}}} > 0, \tag{40}$$

where $\tilde{x}_{i2}$ is the critical threshold of the private signal used by player i, $\hat{x}_2$ is the threshold used by all other investors, and $\hat{\theta}_2(\hat{x}_2)$ is the critical threshold of the fun-

damental in region 2 when $n = 0$. This is because $\Pr\{\theta_2 < \theta_2^* | \theta_1, x_{i2}\}$ is monotonically decreasing in x_{i2}, using a result of Milgrom (1981) (see below). Furthermore, given that all other investors use a threshold strategy, $\Pr\{\theta_2 < \hat{\theta}_2(\hat{x}_2) | \theta_1, x_{i2}\}$ increases in $\hat{x}_2$ (again see below). Following Vives (2005), the best response of player i is to use a threshold strategy with attack threshold $\tilde{x}_{i2}$, where $\Pr\{\theta_2 < \hat{\theta}_2(\hat{x}_2) | \theta_1, \tilde{x}_{i2}\} = \gamma_2$, implying $r' > 0$. Therefore, our focus on monotone equilibria is valid and we determine conditions sufficient for a unique monotone Bayesian equilibrium.

The conditional density function $f(x|\theta)$ is normal with mean θ and satisfies the monotone likelihood ratio property: for all $x_i > x_j$ and $\theta' > \theta$, we have

$$\frac{f(x_i|\theta')}{f(x_i|\theta)} \geq \frac{f(x_j|\theta')}{f(x_j|\theta)} \Leftrightarrow \frac{\phi\left(\sqrt{\beta}(x_i - \theta')\right)}{\phi\left(\sqrt{\beta}(x_i - \theta)\right)} \geq \frac{\phi\left(\sqrt{\beta}(x_j - \theta')\right)}{\phi\left(\sqrt{\beta}(x_j - \theta)\right)}. \tag{41}$$

Using Proposition 1 of Milgrom (1981), we conclude that $\Pr\{\theta_2 \leq \theta_2^* | \theta_1, x_{i2}\}$ monotonically decreases in x_{i2}. Hence, $\frac{d\Pr\{\theta_2 \leq \theta_2^* | \theta_1, \hat{x}_2\}}{d\theta_2^*} > 0$. Equation (33) then implies that

$$0 \leq \frac{d\hat{\theta}_2(\hat{x}_2)}{d\hat{x}_2} \leq \left(1 + \sqrt{\frac{2\pi}{\beta}}\right)^{-1}. \tag{42}$$

Existence and uniqueness

Lemma 2 *Suppose there is a crisis in region* 1, $\theta_1 < \theta_1^*$, *and investors are uninformed about the correlation,* $n = 0$. *If private information is sufficiently precise,* $\beta > \underline{\beta}_1'$, *then there exists a unique monotone Bayesian equilibrium in region* 2. *Each investor attacks if and only if the private signal is below the threshold* x_U^*. *A crisis occurs if and only if the fundamental in region* 2 *is below the fundamental threshold* $\theta_2^*(0,0,\theta_1)$ *defined by equation (33). This fundamental threshold is a weighted average of the thresholds that prevail if investors were informed:*

$$\min\{\theta_2^*(1,0,\theta_1), \theta_2^*(1,\rho_H,\theta_1)\} < \theta_2^*(0,0,\theta_1) < \max\{\theta_2^*(1,0,\theta_1), \theta_2^*(1,\rho_H,\theta_1)\}.$$

Proof The proof is in three steps. First, we show that $J(0,\theta_2,\theta_1) \to 1 > \gamma_2$ as $\theta_2 \to 0$, and $J(0,\theta_2,\theta_1) \to 0 < \gamma_2$ as $\theta_2 \to 1$. Second, we show that $\frac{dJ(0,\theta_2,\theta_1)}{d\theta_2} < 0$ for some sufficiently high but finite values of β, such that J strictly decreases in θ_2. We denote this lower bound as $\underline{\beta}_1'$. Therefore, if θ_2^* exists, it is unique. Third, by continuity, there exists a $\theta_2^*(0,0,\theta_1)$ that solves $J(0,\theta_2,\theta_1) = \gamma_2$.

Step 1 (limiting behavior): Observe that $J(0,\theta_2,\theta_1)$ is a weighted average of $F_2(\theta_2,0)$ and $F_2(\theta_2,\rho_H)$. As $\theta_2 \to 0$, then $F_2(\theta_2,\rho) \to 1$ for any $\rho \in \{0,\rho_H\}$, so $J(0,\theta_2,\theta_1) \to 1 > \gamma_2$. Likewise, as $\theta_2 \to 1$, then $F_2(\theta_2,\rho) \to 0$ for any $\rho \in \{0,\rho_H\}$, so $J(0,\theta_2,\theta_1) \to 0 < \gamma_2$.

Step 2 (strictly negative slope): Using the indifference condition of uninformed investors to substitute x_U^* in equation (33), the total derivative of J is

$$\begin{aligned}\frac{dJ(0,\theta_2,\theta_1)}{d\theta_2} &= \hat{p}(\theta_2)\frac{dF_2(\theta_2,0)}{d\theta_2} + (1-\hat{p}(\theta_2))\frac{dF_2(\theta_2,\rho_H)}{d\theta_2} \\ &+ \frac{d\hat{p}(\theta_1,x_U(\theta_2))}{dx_U}\frac{dx_U(\theta_2)}{d\theta_2}\big[F_2(\theta_2,0)-F_2(\theta_2,\rho_H)\big]. \qquad (43)\end{aligned}$$

The proof proceeds by inspecting the individual terms of equation (43).

We know from our analysis of the case of informed investors that $\frac{dF_2(\theta_2,0)}{d\theta_2} < 0$ if $\beta > \underline{\beta}_0$ and that $\frac{dF_2(\theta_2,\rho_H)}{d\theta_2} < 0$ if $\beta > \underline{\beta}_0'$. Moreover, these derivatives are also strictly negative in the limit when $\beta \to \infty$. Thus, the first two components of the sum are negative and finite in the limit when $\beta \to \infty$. By continuity, these terms are also negative for a sufficiently high but finite private noise.

The sign of the third summand in (43) is ambiguous: $F_2(\theta_2^*(0,0,\theta_1),0) \leq F_2(\theta_2^*(0,\rho_H,\theta_1),\rho_H)$ whenever $\theta_2^*(1,\theta_1,0) \leq \theta_2^*(1,\theta_1,\rho_H)$ and $F_2(\theta_2^*(0,0,\theta_1),0) > F_2(\theta_2^*(0,\rho_H,\theta_1),\rho_H)$ otherwise, where $\theta_2^*(0,0,\theta_1) = \theta_2^*(0,\rho_H,\theta_1)$. However, the difference vanishes in the limit when $\beta \to \infty$.

The last term to consider is $\frac{d\hat{p}(\theta_1,x_U(\theta_2))}{dx_U(\theta_2)}\frac{dx_U}{d\theta_2}$. Given the previous sufficient conditions on the relative precision of the private signal,

$$0 < \frac{dx_U}{d\theta_2} = 1 + \frac{1}{\sqrt{\beta}}\frac{1}{\phi(\Phi^{-1}(\theta_2))} < 1 + \frac{\sqrt{2\pi}}{\alpha}.$$

Finally, from section A.1.2, we know that the sign of $\frac{d\hat{p}}{dx_U}$ is ambiguous. However, the derivative is finite for $\beta \to \infty$. Taken together with the zero limit of the first factor of the third term, this term vanishes in the limit.

As a result, by continuity, there must exist a finite level of precision $\beta > \underline{\beta}_1' \in (0,\infty)$ such that $\frac{dJ(0,\theta_2,\theta_1)}{d\theta_2} < 0$ for all $\beta > \underline{\beta}_1'$. This concludes the second step of the proof and therefore the overall proof of Lemma 2. *(q.e.d.)*

A.2 Proof of Proposition 1

The proof considers two cases. In case 1 we establish the existence of a unique monotone equilibrium after a crisis in region 1, i.e. if $\theta_1 < \mu$. In case 2 we extend the existence and uniqueness result to $\theta_1 \geq \mu$.

Case 1: The subcase of $n = 1$ is trivial, since it is merely a corollary of Lemma 1 (Morris and Shin 2003). In what follows, we consider the case of a given $\theta_1 < \theta_1^*$ and $n < 1$, whereby some investors are uninformed. This proof establishes the conditions sufficient for the existence of a unique pair of fundamental thresholds by analyzing a system characterized by two equations, (38) and (39), in two unknowns, $\theta_2(0)$ and $\theta_2(\rho_H)$. The proof builds heavily on the description of the coordination stage in the case of potentially asymmetrically informed investors described in Appendix A.1. We show the existence and uniqueness of the pair $(\theta_2^*(0), \theta_2^*(\rho_H))$. Then, the signal thresholds are uniquely backed out from $(\theta_2^*(0), \theta_2^*(\rho_H))$.

Outline of proof First, we analyze the relationship between $\theta_2(0)$ and $\theta_2(\rho_H)$ as governed by K. Using equations (38) and (37), $\frac{\partial K}{\partial \theta_2^*(0)} > 0$ and $\frac{\partial K}{\partial \theta_2(\rho_H)} < 0$. Hence, $\frac{d\theta_2(0)}{d\theta_2(\rho_H)} > 0$ by the implicit function theorem.

Second, we analyze the relationship between $\theta_2(0)$ and $\theta_2(\rho_H)$ as governed by L. It can be shown that $\beta > \underline{\beta}_0'$ is sufficient for $\frac{\partial L}{\partial \theta_2(\rho_H)} < 0$. Thus, one can show that $\frac{dL}{d\theta_2(0)} < 0$ holds for a sufficiently high but finite value of β. This is proven by generalizing the argument of the proof of Lemma 2, so $\lim_{\beta \to \infty}[\Psi(\theta_2^*(0), x_U^*, 0) - \Psi(\theta_2^*(\rho_H), x_U^*, \rho_H)] = 0$. Hence, $\frac{d\theta_2(0)}{d\theta_2(\rho_H)} < 0$ in the limit. By continuity, there exists a finite precision, $\beta > \underline{\beta}_1$, of private information that guarantees the inequality as well. Taking both of these points together, $(\theta_2^*(0), \theta_2^*(\rho_H))$ is unique if it exists. This arises from the established strict monotonicity and the opposite sign.

Third, we establish the existence of $(\theta_2^*(0), \theta_2^*(\rho_H))$ by making two points: (i) for the highest permissible value of $\theta_2(0)$, the value of $\theta_2(\rho_H)$ prescribed by K is strictly larger than the value of $\theta_2(\rho_H)$ prescribed by L; and (ii) for the lowest permissible value of $\theta_2(0)$, the value of $\theta_2(\rho_H)$ prescribed by K is strictly smaller than the value of $\theta_2(\rho_H)$ prescribed by L.

Formal argument To make these points, consider the following auxiliary step. For any $\theta_2(\rho) \geq \theta_2^*(1,\rho,\theta_1)$, it can be shown that

$$\frac{\partial}{\partial n}\Phi^{-1}\left(\frac{\theta_2^*(\rho) - n\Phi\left(\frac{\alpha_2(\rho)(\theta_2^*(\rho)-\mu_2(\rho,\theta_1))-\sqrt{\alpha_2(\rho)+\beta}\,\Phi^{-1}(\gamma_2)}{\sqrt{\beta}}\right)}{1-n}\right) \geq 0, \tag{44}$$

because $F_2(\theta_2(\rho),\rho) \leq \gamma_2$ for any $\rho \in \{0,\rho_H\}$. Note that both the previous expression and the partial derivative hold with strict inequality if $\theta_2(\rho) > \theta_2^*(1,\rho,\theta_1)$.

Inspecting the inside of the inverse of the cdf, Φ^{-1}, we define the highest permissible value of $\theta_2(\rho)$ that is labelled $\overline{\theta}_2(\rho,n)$ for all ρ:

$$1 = \frac{\overline{\theta}_2(\rho,n) - n\Phi\left(\frac{\alpha_2(\rho)(\overline{\theta}_2(\rho,n)-\mu_2(\rho,\theta_1))-\sqrt{\alpha_2(\rho)+\beta}\,\Phi^{-1}(\gamma_2)}{\sqrt{\beta}}\right)}{1-n}. \tag{45}$$

Hence, $1 \geq \overline{\theta}_2(\rho,1) \geq \theta_2^*(1,\rho,\theta_1)\ \forall\rho$, where the first (second) inequality binds if and only if $n=0$ ($n=1$).

Next, evaluate K at the highest permissible value, $\theta_2(0) = \overline{\theta}_2(0,n)$, which yields $\theta_2(\rho_H) = \overline{\theta}_2(\rho_H,n)$. Likewise, evaluate L at the highest permissible value, $\theta_2(0) = \overline{\theta}_2(0,n)$, which yields $\theta_2(\rho_H) < \overline{\theta}_2(\rho_H,n)$. This proves point (i).

We proceed with point (ii). We can similarly define the lowest permissible value of $\theta_2(\rho)$, which is labelled $\underline{\theta}_2(\rho,n)$ for all ρ. Then, $0 \leq \underline{\theta}_2(\rho,1) \leq \theta_2^*(1,\rho,\theta_1)\ \forall\rho$, where the first (second) inequality binds if and only if $n=0$ ($n=1$).

Next, evaluate K at the lowest permissible value, $\theta_2(0) = \underline{\theta}_2(0,n)$, which yields $\theta_2(\rho_H) = \underline{\theta}_2(\rho_H,n)$. Likewise, evaluate L at $\theta_2(0) = \underline{\theta}_2(0,n)$, which yields $\theta_2(\rho_H) > \underline{\theta}_2(\rho_H,n)$. This proves point (ii) and completes the proof of case 1.

Case 2: For the case of no crisis in region 1 the fundamental correlation is unobserved, i.e. $n=0$. Lemma 2 establishes the existence and uniqueness of a monotone equilibrium for the case of a crisis in 1 and $n=0$. This result can be extended to the case when $\theta_1 \geq \theta_1^*$ by using the mixture distribution approach. In particular, $J(0,\theta_2,\theta_1) = \gamma_2$ is modified to account for the additional dimension of uncertainty about the state θ_1:

$$\begin{aligned} \hat{p} \equiv\ & f(\rho=0,\theta_1|x_{i2}) \\ =\ & \frac{p\Pr\{x_{i2}|\theta_1,\rho=0\}f(\theta_1|\theta_1\geq\theta_1^*)}{\int_{\theta_1^*}^{+\infty}\left(p\Pr\{x_{i2}|\theta_1,\rho=0\}+(1-p)\Pr\{x_{i2}|\theta_1,\rho=\rho_H\}\right)f(\theta_1|\theta_1\geq\theta_1^*)d\theta_1}, \end{aligned} \tag{46}$$

where the prior density is given by $f(\theta_1|\theta_1 \geq \theta_1^*) = f(\theta_1)/(1-F(\theta_1^*))\ \forall \theta_1 \geq \theta_1^*$. Hence, the modified equilibrium condition reads:

$$\int_{\theta_1^*}^{+\infty} J(0,\theta_2,\theta_1)d\theta_1 = \gamma_2. \tag{47}$$

Following an argument analog to the proof of Lemma 2, we can find the limiting behavior $\int_{\theta_1^*}^{+\infty} J(0,\theta_2,\theta_1)d\theta_1 \to 1 > \gamma_2$ as $\theta_2 \to 0$, and $\int_{\theta_1^*}^{+\infty} J(0,\theta_2,\theta_1)d\theta_1 \to 0 < \gamma_2$ as $\theta_2 \to 1$. Furthermore, $\left(d\int_{\theta_1^*}^{+\infty} J(0,\theta_2,\theta_1)\right)/d\theta_2 < 0$ for some sufficiently high but finite values of β. Hence, if θ_2^* exists, it is unique. Finally, by continuity, there exists a $\theta_2^*(0,0,\theta_1)$ that solves equation (47), completing the proof of case 2. *(q.e.d.)*

B Contagion

B.1 Proof of Lemma 1

Investors are uninformed about the realized correlation ρ, thereby considering the possibilities of both positively correlated and uncorrelated fundamentals. The proof considers two cases about when the realized fundamental θ_1 is observed. In the counterfactual case 1, investors always observe the realized θ_1. In case 2, as assumed in the model, investors only observe θ_1 after a crisis in region 1, $\theta_1 < \mu$. Introducing this counterfactual is helpful for constructing the proof.

Case 1: First, it can be shown, by a direct extension of the proof of Proposition 1, that there exists a unique fundamental threshold $\theta_2^*(n=0,\rho,\theta_1)$ if θ_1 is observed after no crisis in region 1, $\theta_1 \geq \mu$ if $\beta > \underline{\beta}_1 \in (0,\infty)$. This fundamental threshold is computed as a weighted average of $\theta_2^*(1,\rho_H,\theta_1)$ and $\theta_2^*(1,0,\theta_1)$, following the logic of Proposition 1 and its proof.

Second, $\Pr\{\theta_2 \leq \theta_2^*(0,\rho,\theta_1)|\theta_1\}$ is continuous and monotonically decreasing in θ_1 for all $\beta > \underline{\beta}_3'$. To see this, consider equation (43) in the proof of Proposition 1 and inspect its analog $\frac{dJ(0,\theta_2,\theta_1)}{d\theta_1}$. Observe that $\frac{dF_2(\theta_2^*,0)}{d\theta_1} = 0$, $\frac{dF_2(\theta_2^*,\rho_H)}{d\theta_1} < 0$ and $\frac{dx_{2U}(\theta_2)}{d\theta_1} = \frac{dx_{2U}(\theta_2^*)}{d\theta_2}\frac{d\theta_2^*}{d\theta_1}$. Using the same argument as in the proof of Proposition 1, there exists a finite level of precision $\beta > \underline{\beta}_3' \in (0,\infty)$ such that $\frac{dJ(0,\theta_2,\theta_1)}{d\theta_1} < 0$ and

$$\frac{d\theta_2^*(0,\rho,\theta_1)}{d\theta_1} = -\left(\frac{dJ(0,\theta_2,\theta_1)}{d\theta_1}\right)/\left(\frac{dJ(0,\theta_2,\theta_1)}{d\theta_2}\right) < 0. \tag{48}$$

This direct effect is exacerbated by an indirect effect via the conditional distribution of $\theta_2|\theta_1$. That is, the left-hand side of (10) is a weighted average over a less-favorable set of values of θ_1 than the right-hand side, with strictly positive weights on each θ_1. Hence, inequality (10) holds for case 1.

Case 2: From case 1, the ranking of fundamental thresholds when θ_1 is observed is $\Pr\{\theta_2 \leq \theta_2^*(0,\rho,\theta_1)|\theta_1 < \theta_1^*\} > \Pr\{\theta_2 \leq \theta_2^*(0,\rho,\theta_1)|\theta_1 = \theta_1^*\} \geq \Pr\{\theta_2 \leq \theta_2^*(0,\rho,\theta_1)|\theta_1 \geq \theta_1^*\}$. This ranking prevails if θ_1 is unobserved in the absence of a crisis in region 1, since the right-hand side of condition (10) is a weighted average over more-favorable values of θ_1. As a result, inequality (10) holds for sufficiently precise private information, where $\underline{\beta}_3 < \infty$ denotes the maximum of the stated lower bounds on the precision of private information. *(q.e.d.)*

B.2 Definition 1 and its implications for the comparative statics

Section B.2.1 derives the conditions for a strong prior, while section B.2.2 presents the implications for the comparative statics.

B.2.1 Constructing Definition 1

This definition allows us to distinguish between weak and strong priors about the fundamental. $X(\rho)$ and $Y(\rho)$ are derived by reformulating equation (26):

$$\Phi^{-1}(\theta_2^*(1,\rho,\theta_1)) - \frac{\alpha_2(\rho)}{\sqrt{\beta}}(\theta_2^*(1,\rho,\theta_1) - \mu_2(\rho,\theta_1)) = -\frac{\sqrt{\alpha_2(\rho)+\beta}}{\sqrt{\beta}}\Phi^{-1}(\gamma_2). \tag{49}$$

First, $X(\rho)$ can be derived by setting $\theta_2^*(1,\rho,\theta_1) = \mu_2(\rho,\theta_1)$ and by isolating $\mu_2(\rho,\theta_1)$. A sufficient condition that assures that strong (weak) prior beliefs are associated with a low (high) incidence of attacks below (above) 50% is derived from equation (49) by setting $\theta_2^* = \frac{1}{2}$. This leads to $Y(\rho)$.

B.2.2 Comparative statics: the precision of public and private information

The following discussion draws in part on Bannier and Heinemann (2005). We have the following partial derivatives of the fundamental thresholds:

$$\frac{d\theta_2^*(1,\rho,\theta_1)}{d\alpha}\begin{cases} < & 0 \; if \; \theta_2^*(1,\rho,\theta_1) < \mu_2(\rho,\theta_1) + \frac{1}{2\sqrt{\alpha_2(\rho)+\beta}}\Phi^{-1}(\gamma_2) \\ \geq & 0 \; otherwise \end{cases}$$

$$\frac{d\theta_2^*(1,\rho,\theta_1)}{d\beta}\begin{cases} > & 0 \; if \; \theta_2^*(1,\rho,\theta_1) < \mu_2(\rho,\theta_1) + \frac{1}{\sqrt{\alpha_2(\rho)+\beta}}\Phi^{-1}(\gamma_2) \\ \leq & 0 \; otherwise. \end{cases}$$

If $b_2 \leq \ell_2$, then a strong prior about the fundamental, $\theta_2^*(1,\rho,\theta_1) < \mu_2(\rho,\theta_1) \; \forall \, \rho \in \{0,\rho_H\}$, implies that $\frac{d\theta_2^*}{d\alpha} < 0$ and $\frac{d\theta_2^*}{d\beta} > 0$. If $b_2 > \ell_2$, then a weak prior, $\theta_2^*(1,\rho,\theta_1) > \mu_2(\rho,\theta_1) \; \forall \, \rho \in \{0,\rho_H\}$, implies that $\frac{d\theta_2^*}{d\alpha} > 0$ and $\frac{d\theta_2^*}{d\beta} < 0$.

Instead, if $b_2 > \ell_2$, then $\theta_2^*(1,\rho,\theta_1) < \mu_2(\rho,\theta_1) \; \forall \, \rho \in \{0,\rho_H\}$ does not necessarily imply that $\frac{d\theta_2^*}{d\alpha} < 0$ and $\frac{d\theta_2^*}{d\beta} > 0$. In other words, the inequalities involving $X(\rho)$ in Definition 1 are no longer sufficient if $b_2 > \ell_2$. However, Definition 1 provides a more-restrictive definition of a strong (weak) prior about fundamentals by imposing additional conditions involving $Y(\rho)$, which assure that a strong (weak) prior is associated with a low (high) incidence of crises below (above) 50%. Hence, Definition 1 also ensures that a strong prior implies that $\frac{d\theta_2^*}{d\alpha} < 0$ and $\frac{d\theta_2^*}{d\beta} > 0$ even if $b_2 > \ell_2$. Similarly, it ensures that a weak prior implies that $\frac{d\theta_2^*}{d\alpha} > 0$ and $\frac{d\theta_2^*}{d\beta} < 0$ even if $b_2 \leq \ell_2$.

Finally, irrespective of the strength of the prior, we have $\frac{d\theta_2^*}{d\mu_2} < 0$.

B.3 Wake-up-call contagion in isolation

As a preliminary of the proof of Proposition 2 this section first analyzes the fundamental threshold ranking in section B.3.1. Then, section B.3.2 establishes the monotonicity of region 2's fundamental thresholds in n for all $\theta_1 < \theta_1^*$. Next, we develop the proof of Proposition 2 in section B.3.3.

B.3.1 Fundamental threshold ranking

Lemma 3 ***Ranking of fundamental thresholds.*** *Suppose private information is sufficiently precise and investors are informed, $n = 1$. After no crisis in region 1, the fundamental threshold ranking $\theta_2^*(1,0,\theta_1) > (\theta_2^*|\theta_1 \geq \theta_1^*)$ is guaranteed to hold. In contrast, after a crisis, the threshold ranking $\theta_2^*(1,0,\theta_1) > \theta_2^*(1,\rho_H,\theta_1)$ is ensured by a strong prior about the fundamental in region 2 and an intermediate level of the realized fundamental in region 1, $\theta_1 \in (\underline{\theta}_1, \mu)$, where the lower bound is defined in Proposition 3. Furthermore, $\theta_2^*(1,0,\theta_1) < \theta_2^*(1,\rho_H,\theta_1) \; \forall \; \theta_1 < \underline{\theta}_1$.*

Proof The proof proceeds by first analyzing the case when a crisis is observed in region 1. The threshold fundamental $\theta_2^* = \theta_2^*(n=1,\rho,\theta_1)$ is implicitly defined by equation (26). For sufficiently precise private information, $\beta > \underline{\beta}_0' \leq \underline{\beta}_1$, $F_2(\theta_2^*,\rho)$ decreases in θ_2^* for a given ρ. Hence, the ranking is $\theta_2^*(1,0,\theta_1) > \theta_2^*(1,\rho_H,\theta_1)$ if $F_2(\theta_2^*(1,0,\theta_1),0) > F_2(\theta_2^*(1,0,\theta_1),\rho_H)$, where $\alpha_2(0) = \alpha$ and $\mu_2(0,\theta_1) = \mu$:

$$\frac{\alpha}{\sqrt{\alpha+\beta}}[\theta_2^*(1,0,\theta_1) - \mu] - \sqrt{\frac{\beta}{\alpha+\beta}}\Phi^{-1}(\theta_2^*(1,0,\theta_1)) > \tag{50}$$
$$\frac{\alpha_2(\rho_H)}{\sqrt{\alpha_2(\rho_H,\theta_1)+\beta}}[\theta_2^*(1,0,\theta_1) - \mu_2(\rho_H,\theta_1)] - \sqrt{\frac{\beta}{\alpha_2(\rho_H)+\beta}}\Phi^{-1}(\theta_2^*(1,0,\theta_1)).$$

Solving for θ_1, which is implicit in $\mu_2(\rho_H,\theta_1)$, results in the lower bound on θ_1, which is defined in equation (15).

Next, $\underline{\theta}_1 < \mu$ arises because, first, $\theta_2^* < \mu$, second, $[1 - \frac{\alpha_2}{\alpha_2(\rho_H)}\sqrt{\frac{\alpha_2(\rho_H)+\beta}{\alpha_2+\beta}}] > 0$ and, third, $\left[\sqrt{\frac{\alpha_2(\rho_H)+\beta}{\alpha_2+\beta}} - 1\right] > 0$. Finally, $\Phi^{-1}(\theta_2^*(1,0,\theta_1)) < 0$ if $\mu_2(\rho,\theta_1) < Y(\rho) \; \forall \rho \in \{0,\rho_H\}$. Hence, $\theta_1 \in [\underline{\theta}_1, \mu]$ is non-empty and the inequality in Lemma 3 follows. (As an aside, if the definition of strong and weak priors used only X, and not also Y, then $[\underline{\theta}_1, \mu]$ may be empty under some parameter values.) This concludes the case when a crisis in region 1 is observed.

If no crisis in region 1 is observed, then it follows that $(\theta_2^*|\theta_1 \geq \theta_1^*) < \theta_2^*(1,0,\theta_1)$ applying the argument of case 2 in the proof of Lemma 1. *(q.e.d.)*

B.3.2 Monotonicity in n

Lemma 4 ***Proportion of informed investors and fundamental thresholds.*** *Suppose there is a crisis in region 1, $\theta_1 < \theta_1^*$, and strong fundamentals in region 2.*

If private information is sufficiently precise, $\underline{\beta} < \beta < \infty$, and public information is sufficiently imprecise, $0 < \alpha < \overline{\alpha}$, then:

(A) Boundedness. *The fundamental thresholds in the polar case of informed investors bound the fundamental thresholds in the general case of asymmetrically informed investors:*

$$\begin{aligned} &if\ \theta_1 \geq \underline{\theta}_1: \quad \theta_2^*(1,\rho_H,\theta_1) \leq \theta_2^*(n,\rho,\theta_1) \leq \theta_2^*(1,0,\theta_1) \ \ \forall \rho \in \{0,\rho_H\} \ \ \forall n \in [0,1] \\ &if\ \theta_1 < \underline{\theta}_1: \quad \theta_2^*(1,0,\theta_1) \leq \theta_2^*(n,\rho,\theta_1) \leq \theta_2^*(1,\rho_H,\theta_1) \ \ \forall \rho \in \{0,\rho_H\} \ \ \forall n \in [0,1]. \end{aligned}$$

(B) Monotonicity. *The fundamental threshold in the case of zero (positive) cross-regional correlation increases (decreases) in the proportion of informed investors. Strict monotonicity is attained if and only if the fundamental thresholds are strictly bounded, that is $\forall \rho, n \in [0,1)$:*

$$\frac{d\theta_2^*(n,0,\theta_1)}{dn} \begin{cases} >0 \ \ if\ \theta_2^*(1,\rho_H,\theta_1) < \theta_2^*(n,\rho,\theta_1) < \theta_2^*(1,0,\theta_1) \\ <0 \ \ if\ \theta_2^*(1,0,\theta_1) < \theta_2^*(n,\rho,\theta_1) < \theta_2^*(1,\rho_H,\theta_1) \\ =0 \ \ if\ \theta_{2I}^*(\rho,\theta_1) = \theta_2^*(n,\rho,\theta_1) \end{cases} \tag{51}$$

$$\frac{d\theta_2^*(n,\rho_H,\theta_1)}{dn} \begin{cases} <0 \ \ if\ \theta_2^*(1,\rho_H,\theta_1) < \theta_2^*(n,\rho,\theta_1) < \theta_2^*(1,0,\theta_1) \\ >0 \ \ if\ \theta_2^*(1,0,\theta_1) < \theta_2^*(n,\rho,\theta_1) < \theta_2^*(1,\rho_H,\theta_1) \\ =0 \ \ if\ \theta_2^*(1,\rho,\theta_1) = \theta_2^*(n,\rho,\theta_1). \end{cases} \tag{52}$$

(C) Monotonicity in signal thresholds. *As a consequence of the monotonicity in fundamental thresholds:*

$$\frac{d|x_I^*(n,0,\theta_1) - x_I^*(n,\rho_H,\theta_1))|}{dn} \geq 0 \ \ \forall\, n \in [0,1). \tag{53}$$

Proof We prove the results of Lemma 4 in turn. A general observation is that the updated belief on the probability of positive cross-regional correlation becomes degenerate: $\hat{p} \dashrightarrow p$ for $\alpha \to 0$. Results (A) and (B) are closely linked, so we start by proving them below.

Results (A) and (B). This proof has three steps.

Step 1: We show in the first step that both fundamental thresholds in the case of asymmetrically informed investors lie either within these bounds or outside of them. As a consequence of $\hat{p} \to p$, condition $L(n,\theta_2^*(0),\theta_2^*(\rho_H)) = 0$ prescribes that, for any n, the thresholds $\theta_2^*(0)$ and $\theta_2^*(\rho_H)$ are either simultaneously within or

outside of the two bounds given by the fundamental thresholds if all investors are informed, $\theta_2^*(1,0,\theta_1)$ and $\theta_2^*(1,\rho_H,\theta_1)$. This is proven by contradiction. First, suppose that $\theta_2^*(\rho_H) < \theta_2^*(1,\rho_H,\theta_1)$ and $\theta_2^*(0) < \theta_2^*(1,0,\theta_1)$. This leads to a violation of $L(\cdot) = 0$ because $J(n,\theta_2^*(0),\theta_2^*(\rho_H)) > \gamma_2 \; \forall \, n$ if $\alpha \to 0$. Second, suppose that $\theta_2^*(\rho_H) > \theta_2^*(1,\rho_H,\theta_1)$ and $\theta_2^*(0) > \theta_2^*(1,0,\theta_1)$. Again, this leads to a violation because $J(n,\theta_2^*(0),\theta_2^*(\rho_H)) < \gamma_2 \; \forall \, n$ if $\alpha \to 0$.

Step 2: We next obtain the derivatives of the fundamental thresholds with respect to the proportion of informed investors, $\frac{d\theta_2^*(\rho)}{dn}$ and $\frac{dx_{2I}^*(\rho)}{dn}$. Applying the implicit function theorem for simultaneous equations, we obtain the following derivatives:

$$\frac{d\theta_2^*(n,0,\theta_1)}{dn} = \frac{\begin{vmatrix} -\frac{\partial K}{\partial n} & \frac{\partial K}{\partial \theta_2(n,\rho_H,\theta_1)} \\ -\frac{\partial L}{\partial n} & \frac{\partial L}{\partial \theta_2(n,\rho_H,\theta_1)} \end{vmatrix}}{\begin{vmatrix} \frac{\partial K}{\partial \theta_2(n,0,\theta_1)} & \frac{\partial K}{\partial \theta_2(n,\rho_H,\theta_1)} \\ \frac{\partial L}{\partial \theta_2(n,0,\theta_1)} & \frac{\partial L}{\partial \theta_2(n,\rho_H,\theta_1)} \end{vmatrix}} \equiv \frac{|M_1|}{|M|}, \tag{54}$$

where $|M| \equiv \det(M)$. We also find that

$$\frac{d\theta_2^*(n,\rho_H,\theta_1)}{dn} = \frac{\begin{vmatrix} \frac{\partial K}{\partial \theta_2(n,0,\theta_1)} & -\frac{\partial K}{\partial n} \\ \frac{\partial L}{\partial \theta_2(n,0,\theta_1)} & -\frac{\partial L}{\partial n} \end{vmatrix}}{\begin{vmatrix} \frac{\partial K}{\partial \theta_2(n,0,\theta_1)} & \frac{\partial K}{\partial \theta_2(n,\rho_H,\theta_1)} \\ \frac{\partial L}{\partial \theta_2(n,0,\theta_1)} & \frac{\partial L}{\partial \theta_2(n,\rho_H,\theta_1)} \end{vmatrix}} \equiv \frac{|M_2|}{|M|}. \tag{55}$$

To find $|M|$, recall from the proof of Proposition 1 that $\frac{\partial K}{\partial \theta_2(0)} > 0$ and $\frac{\partial K}{\partial \theta_2(\rho_H)} < 0$. Furthermore, $\frac{\partial L}{\partial \theta_2(\rho_H)} < 0$ and $\frac{\partial L}{\partial \theta_2(0)} < 0$ for a sufficiently high but finite value of β. As a result, $|M| < 0$ for a sufficiently high but finite value of β.

The proof proceeds by analyzing $|M_1|$ and $|M_2|$. To do this, we first examine the derivatives $\frac{\partial K}{\partial n}$ and $\frac{\partial L}{\partial n}$. Thereafter, we combine the results to obtain the signs of $|M_1|$ and $|M_2|$. We obtain $\forall n \in [0,1)$:

$$\frac{\partial K}{\partial n} = \begin{cases} < 0 \;\; if \; \theta_2^*(n,0,\theta_1) < \theta_2^*(1,0,\theta_1) \wedge \theta_2^*(n,\rho_H,\theta_1) > \theta_2^*(1,\rho_H,\theta_1) \\ > 0 \;\; if \; \theta_2^*(n,0,\theta_1) > \theta_2^*(1,0,\theta_1) \wedge \theta_2^*(n,\rho_H,\theta_1) < \theta_2^*(1,\rho_H,\theta_1) \\ = 0 \;\; if \; \theta_2^*(n,0,\theta_1) = \theta_2^*(1,0,\theta_1) \wedge \theta_2^*(n,\rho_H,\theta_1) = \theta_2^*(1,\rho_H,\theta_1). \end{cases}$$

After having found the partial derivative for one equilibrium condition (K), we turn to the other equilibrium condition (L). Here, we can invoke the envelope theorem in order to obtain $\frac{\partial L}{\partial n} = 0$. The idea is the following. Since L represents the indifference condition of an uninformed investor, the proportion of informed investors enters only indirectly via x^*_{2U} and we can write:

$$\frac{\partial L}{\partial n} = \frac{\partial J}{\partial x^*_{2U}} \frac{\partial x^*_{2U}}{\partial n} + \overbrace{\frac{\partial J}{\partial n}}^{=0}. \tag{56}$$

Since x^*_{2U} is the optimal signal threshold of an uninformed investor, it satisfies $J(\cdot, x^*_{2U}) = \gamma_2$. Thus, we must have $\frac{\partial J}{\partial x^*_{2U}} = 0$, which corresponds to a first-order optimality condition. (This implicitly uses the result that the equilibrium is unique.)

Third, we obtain the derivatives of the fundamental thresholds for sufficiently small but positive values of α. We find that $\forall n \in [0,1)$:

$$\frac{d\theta^*_2(n,0,\theta_1)}{dn} = \begin{cases} > 0 \;\; if\; \theta^*_2(n,0,\theta_1) < \theta^*_2(1,0,\theta_1) \wedge \theta^*_2(n,\rho_H,\theta_1) > \theta^*_2(1,\rho_H,\theta_1) \\ < 0 \;\; if\; \theta^*_2(n,0,\theta_1) > \theta^*_2(1,0,\theta_1) \wedge \theta^*_2(n,\rho_H,\theta_1) < \theta^*_2(1,\rho_H,\theta_1) \\ = 0 \;\; if\; \theta^*_2(n,0,\theta_1) = \theta^*_2(1,0,\theta_1) \wedge \theta^*_2(n,\rho_H,\theta_1) = \theta^*_2(1,\rho_H,\theta_1), \end{cases}$$

and $\forall n \in [0,1)$:

$$\frac{d\theta^*_2(n,\rho_H,\theta_1)}{dn} = \begin{cases} < 0 \;\; if\; \theta^*_2(n,0,\theta_1) < \theta^*_2(1,0,\theta_1) \wedge \theta^*_2(n,\rho_H,\theta_1) > \theta^*_2(1,\rho_H,\theta_1) \\ > 0 \;\; if\; \theta^*_2(n,0,\theta_1) > \theta^*_2(1,0,\theta_1) \wedge \theta^*_2(n,\rho_H,\theta_1) < \theta^*_2(1,\rho_H,\theta_1) \\ = 0 \;\; if\; \theta^*_2(n,0,\theta_1) = \theta^*_2(1,0,\theta_1) \wedge \theta^*_2(n,\rho_H,\theta_1) = \theta^*_2(1,\rho_H,\theta_1). \end{cases}$$

Step 3: In this final step, we combine the results from the previous two steps to show both boundedness and monotonicity. In particular, we use the result that the derivative of the fundamental threshold with respect to the proportion of informed investors is zero once the boundary is hit. Therefore, the thresholds in the general case of asymmetrically informed investors are always bounded, which proves

Result (A). The distinction between the two cases arises because:

$$\theta_2^*(1,0,\theta_1) = \begin{cases} > \theta_2^*(1,\rho_H,\theta_1) & if\ \theta_1 > \underline{\theta}_1 \\ < \theta_2^*(1,\rho_H,\theta_1) & if\ \theta_1 < \underline{\theta}_1 \\ = 0 & if\ \theta_1 = \underline{\theta}_1. \end{cases} \tag{57}$$

Given boundedness, in turn, the derivatives of the fundamental threshold can be clearly signed, yielding Result (B).

Next, for the case of $\theta_1 \geq \underline{\theta}_1$, we prove that $\theta_2^*(1,\rho_H,\theta_1) \leq \theta_2^*(\rho_H), \theta_2^*(0) \leq \theta_2^*(1,0,\theta_1)$ for all n if α is sufficiently small. First, $\theta_2^*(1,\rho_H,\theta_1) < \theta_2^*(0) = \theta_2^*(\rho_H) = \theta_2^*(0,\rho,\theta_1) < \theta_2^*(1,0,\theta_1)$ if $n=0$, while $\theta_2^*(0) = \theta_2^*(1,0,\theta_1)$ and $\theta_2^*(\rho_H) = \theta_2^*(1,\rho_H,\theta_1)$ if $n=1$. Second, $\frac{d\theta_2^*(0)}{dn}\big|_{n=0} > 0$ and $\frac{d\theta_2^*(0)}{dn}\big|_{n=1} = 0$. Third, by continuity $\theta_2^*(0,\rho,\theta_1) < \theta_2^*(0) < \theta_2^*(1,0,\theta_1)$ and $\frac{d\theta_2^*(n,\theta_1,0)}{dn} > 0$ for small values of n. Fourth, if for any $\hat{n} \in (0,1]$ $\theta_2^*(0) \nearrow \theta_2^*(1,0,\theta_1)$ when $n \to \hat{n}$, then – for sufficiently small but positive values of α – it has to be true that $\theta_2^*(\rho_H) \searrow \theta_2^*(1,\rho_H,\theta_1)$ when $n \to \hat{n}$. This is because of the result in *step 1*. Fifth, given $\frac{d\theta_2^*(n,0,\theta_1)}{dn} < 0$ if $\theta_2^*(0) > \theta_2^*(1,0,\theta_1)$ and $\theta_2^*(\rho_H) < \theta_2^*(1,\rho_H,\theta_1)$, it follows by continuity that $\theta_2^*(0) = \theta_2^*(1,0,\theta_1)$ and $\theta_2^*(\rho_H) = \theta_2^*(1,\rho_H,\theta_1)$ for all $n \geq \hat{n}$. In conclusion, $\theta_2^*(1,\rho_H,\theta_1) \leq \theta_2^*(\rho_H), \theta_2^*(0) \leq \theta_2^*(1,0,\theta_1)$ for all $n \in [0,1]$ if α is sufficiently small.

For the case $\theta_1 < \underline{\theta}_1$, it can be proven that $\theta_2^*(1,\rho_H,\theta_1) \geq \theta_2^*(\rho_H), \theta_2^*(0) \geq \theta_2^*(1,0,\theta_1)\ \forall\, n$ if α is sufficiently small using a similar argument (all signs in relation to fundamental thresholds flip).

Result (C). From equation (35),

$$x_I^*(\rho) = \theta_2^*(\rho) + \frac{\theta_2^*(\rho) - \mu_2(\rho,\theta_1)}{\alpha_2(\rho,\theta_1)^{-1}\beta} - \frac{\sqrt{\alpha_2(\rho,\theta_1)+\beta}}{\beta}\Phi^{-1}(\gamma_2) \tag{58}$$

$$\Rightarrow \frac{dx_I^*(\rho)}{dn} = \frac{d\theta_2^*(\rho)}{dn}\left(\frac{\beta}{\alpha_2(\rho,\theta_1)+\beta}\right)^{-1}. \tag{59}$$

Therefore, by continuity, there exists a sufficiently small but positive value of α that implies the required inequality, taking into account the monotonicity of the fundamental thresholds. Therefore, the distance between the fundamental thresholds is monotone for any $n > 0$, which implies $\frac{d|(x_I^*(0) - x_I^*(\rho_H))|}{dn} > 0$. *(q.e.d.)*

B.3.3 Proof of Proposition 2

After a crisis in region 1, $\theta_1 < \mu$, all investors observe the realized θ_1 and a proportion n of investors observe the realized correlation ρ. Consistent with our previous notation, $\theta_2^*(n=0,\rho,\theta_1) \equiv \theta_2^*|\theta_1 \geq \theta_1^*$ denotes the fundamental threshold of region 2 after no crisis in region 1 and $\theta_2^*(n,\rho,\theta_1) \equiv \theta_2^*|\theta_1 < \theta_1^*, n$ after a crisis.

The proof builds on Lemma 3 and is constructed in four steps. First, we decompose the right-hand side of equation (13) for $E_3 \equiv \theta_2 < \theta_2^*(0,\rho,\theta_1)$ by the law of total probability, $\Pr\{E_3|\theta_1 \geq \theta_1^*\} = p\Pr\{E_3|\rho=0,\theta_1 \geq \theta_1^*\} + (1-p)\Pr\{E_3|\rho = \rho_H, \theta_1 \geq \theta_1^*\}$. Since $p \in (0,1)$, it then suffices to show both of the following inequalities:

$$\Pr\{\theta_2 < \theta_2^*(n,0,\theta_1)|\rho=0,\theta_1<\theta_1^*\} > \Pr\{\theta_2 < \theta_2^*(0,\rho,\theta_1)|\rho=0,\theta_1\geq\theta_1^*\} \quad (60)$$

$$\Pr\{\theta_2 < \theta_2^*(n,0,\theta_1)|\rho=0,\theta_1<\theta_1^*\} > \Pr\{\theta_2 < \theta_2^*(0,\rho,\theta_1)|\rho=\rho_H,\theta_1\geq\theta_1^*\}, \quad (61)$$

for all $n \in [0,1]$, which we do below. In other words, we construct sufficient conditions without resorting to the ex-ante probability of a positive correlation.

Second, we consider the case of $n=0$. It can be shown, by a direct extension of Proposition 1, that there exists a unique $\theta_2^*(n=0,\rho,\theta_1)$ after no crisis in region 1 (see the proof of Lemma 1). Given that the true distribution of θ_2 is the same on both sides of inequality (60), the result follows directly. We have that $\theta_2^*|\theta_1 \geq \theta_1^*$ must be strictly smaller than $\theta_2^*(n=0,0,\theta_1<\theta_1^*)$, since the former consists of a weighted average of the fundamental thresholds $\theta_2^*(n=0,0,\theta_1)$ for each $\theta_1 \geq \theta_1^*$ with strictly positive weight on each $\theta_1 \geq \theta_1^*$. For inequality (61), observe that θ_2 is drawn from a more-favorable distribution if $\rho = \rho_H$ because $\theta_1 \geq \theta_1^* = \mu$, which works for our result. Hence, inequality (61) is guaranteed to hold for $n=0$.

Third, consider the case of $n=1$. Recall that $[\theta_2^*(n=0,\rho,\theta_1)|\theta_1 \geq \theta_1^*]$ is a weighted average of $\theta_2^*(n=1,\rho_H,\theta_1)$ and $\theta_2^*(n=1,0,\theta_1)$ with strictly positive weights. Since $\theta_2^*(n=1,\rho_H,\theta_1) < \theta_2^*(n=1,0,\theta_1)$ for all $\theta_1 > \underline{\theta}_1$ (Lemma 3) and, hence, for all $\theta_1 \geq \theta_1^*$, we have that $\theta_2^*(n=1,0,\theta_1) > [\theta_2^*(n=0,\rho,\theta_1)|\theta_1 \geq \theta_1^*]$. Hence, inequality (60) holds. Given that θ_2 is drawn from a more-favorable distribution if $\rho = \rho_H$, inequality (61) is guaranteed to hold.

Fourth, consider the case of $n \in (0,1)$. Recall from Lemma 4 that $\theta_2^*(n,0,\theta_1)$ is continuous and strictly monotone in n for $n \in (0,1)$. Hence, (60) and (61) hold for

all $n \in [0,1]$. As a result, inequality (13) holds for sufficiently precise private information, where $\underline{\beta}_4 < \infty$ denotes the maximum of the lower bounds on the precision of private information. *(q.e.d.)*

B.4 Comparative statics and fundamental threshold ranking

This section analyzes the interaction between the *mean effect* and the *variance effect*. This interaction determines the ordering of fundamental thresholds $\theta_2^*(1,0,\theta_1)$ and $\theta_2^*(1,\rho_{II},\theta_1)$. However, note that our focus here is only on the ordering of fundamental thresholds, and not on the ordering of probability of a crisis. There is no one-to-one mapping between the ordering of fundamental thresholds and the ordering of the probability of a crisis, since the realized correlation also affects the conditional distribution of the fundamental, $\theta_2|\rho$.

Metz (2002) was one of the first to examine the dependence of the fundamental threshold on the precision of private and public information (β, α). An inspection of equation (26) for the special case $b_2 = \ell_2$ reveals that the fundamental threshold $\theta_2^*(1,0,\theta_1)$ increases (decreases) in the precision of the private signal β when the prior is strong (weak). This result is consistent with the findings of Rochet and Vives (2004). A related result is that the above relationship is opposite when considering a change in the precision of the public signal α.

Table B.1 summarizes the effects of an increase in the correlation ρ if $\theta_1 < \mu$. This affects both the mean $\mu_2(\rho,\theta_1)$ and the precision $\alpha_2(\rho)$ of the updated prior about θ_2. The effect of an increase in ρ on $\theta_2^*(1,\rho,\theta_1)$, and its impact on the ranking of fundamental thresholds, depends on the strength of the prior. The cases where the mean effect (ME) and the variance effect (VE) go in opposite directions are shown in bold in Table B.1. For a potentially positive correlation, this requires a strong prior.

To understand the mechanics behind the results in Table B.1, recall that $\frac{d\alpha_2(\rho)}{d|\rho|} > 0$. As a result, the precision of the public signal is lowest when fundamentals are uncorrelated, $\alpha < \alpha_2(\rho_{II})$. Hence, the variance effect tends to decrease (increase) $\theta_2^*(1,\rho,\theta_1)$ if the prior belief is that fundamentals are strong (weak). Thus, for a strong prior, there is a tension between the mean and the variance effect if $\rho_{II} > 0$. This tension is crucial for Lemma 3, derived below. By contrast, after no crisis in

Prior belief	Effect of an increase in ρ on $\theta_2^*(1,\rho,\theta_1)$		Ordering of thresholds	
	Mean effect $\frac{d\theta_2^*(1,\rho,\theta_1)}{d\mu_2}\frac{d\mu_2(\rho,\theta_1)}{d\rho}$	Variance effect $\frac{d\theta_2^*(1,\rho,\theta_1)}{d\alpha_2}\frac{d\alpha_2(\rho)}{d\lvert\rho\rvert}$	$\rho_H > 0$	$\rho_H < 0$
strong	> 0 $\forall$ $\rho \in (-1,1)$	< 0	$\theta_2^*(1,\rho_H,\theta_1) < \theta_2^*(1,0,\theta_1)$ **if VE > ME**	$\theta_2^*(1,\rho_H,\theta_1) < \theta_2^*(1,0,\theta_1)$
weak		> 0	$\theta_2^*(1,\rho_H,\theta_1) > \theta_2^*(1,0,\theta_1)$	$\theta_2^*(1,\rho_H,\theta_1) > \theta_2^*(1,0,\theta_1)$ **if VE > ME**

Table B.1: Effect of an increase in ρ on the ordering of the fundamental threshold in region 2 when all investors are informed after a crisis in region 1, $\theta_1 < \theta_1^* = \mu$.

region 1, $\theta_1 \geq \mu$, there is no tension between the mean and variance effects, since they go in the same direction. We use this last result in the proof of Lemma 1.

Threshold ranking Investors in region 2 reassess the local fundamental θ_2 when learning about a positive correlation. Both the mean and the variance of the updated prior about θ_2 are lower after a crisis in region 1 (see Figure 1). Therefore, the relative size of these mean and variance effects determines the overall impact on the fundamental threshold relative to the case of a zero correlation, $\theta_2^*(1,\rho_H,\theta_1) \lesseqgtr \theta_2^*(1,0,\theta_1)$.[19] We establish conditions for a ranking of thresholds after a crisis in region 1, specifically the sufficient conditions stated in Lemma 3.

At the core of Lemma 3 is the variance of the updated prior and its dependence on the realized correlation. As just derived in Table B.1, the variance effect opposes the mean effect for a strong prior. To limit the size of the mean effect, we require a lower bound $\underline{\theta}_1$ to ensure that the variance effect dominates the mean effect, thereby generating the ranking $\theta_2^*(1,0,\theta_1) > \theta_2^*(1,\rho_H,\theta_1)\ \forall\ \theta_1 \in (\underline{\theta}_1,\mu)$. A decrease in the relative precision of public signals due to a lower realized ρ increases the disagreement between informed investors, which induces them to attack more aggressively. Note that the ranking reverses for low realized θ_1, $\theta_2^*(1,0,\theta_1) < \theta_2^*(1,\rho_H,\theta_1)\ \forall\ \theta_1 < \underline{\theta}_1$. (See also the proof of Lemma 4.)

[19]The ranking of fundamental thresholds does not map one-to-one into a ranking of the probability of a crisis in region 2. The distribution of θ_2 conditional on θ_1 varies with the correlation of regional fundamentals. In particular, the distribution of $\theta_2|\rho = \rho_H, \theta_1$ places greater weight on lower realizations than the distribution of $\theta_2|\rho = 0, \theta_1$.

B.5 Proof of Proposition 3

The proof has five steps. First, consider the symmetric information cases of $n = 0$ and $n = 1$. Then, $\beta > \max\{\underline{\beta}_0', \underline{\beta}_1\} < \infty$ meets the sufficient conditions of Proposition 1, so $\theta_2^*(1, \rho, \theta_1)$ and $\theta_2^*(0, \rho, \theta_1)$ are unique. Second, we have the threshold ranking $\theta_2^*(1, 0, \theta_1) > \theta_2^*(1, \rho_H, \theta_1)$ under the sufficient conditions of Lemma 3; that is, an intermediate realized fundamental in region 1, $\theta_1 \in (\underline{\theta}_1, \mu]$, and a strong prior about the fundamental in region 2 (Definition 1).

Third, Proposition 1 implies that the fundamental threshold when all investors are uninformed, $\theta_2^*(0, \rho, \theta_1)$, is a weighted average of the fundamental thresholds used by informed investors. Since the weight satisfies $\hat{p} \in (0, 1)$, we have the following ranking:

$$\min\{\theta_2^*(1, 0, \theta_1), \theta_2^*(1, \rho_H, \theta_1)\} < \theta_2^*(0, \rho, \theta_1) < \max\{\theta_2^*(1, 0, \theta_1), \theta_2^*(1, \rho_H, \theta_1)\}.$$

Combined with the second point, we have $\theta_2^*(1, 0, \theta_1) > \theta_2^*(0, \rho, \theta_1) \; \forall \; \theta_1 \in (\underline{\theta}_1, \mu]$.

Fourth, given that the realized correlation of regional fundamentals is zero, $\rho = 0$, the ordering of thresholds implies an ordering of probabilities. That is, the probability of a crisis in region 2 is higher when all investors are informed than when all investors are uninformed:

$$Pr\{\theta_2 < \theta_2^*(n = 1, \rho = 0, \theta_1)\} > Pr\{\theta_2 < \theta_2^*(n = 0, \rho = 0, \theta_1)\}, \forall \; \theta_1 \in (\underline{\theta}_1, \mu].$$

Fifth, we generalize the result to any proportion of informed investors, $n \in (0, 1)$, which yields the result stated in equation (14). From Lemma 4, we have $\frac{d\theta_2^*(n, \rho = 0, \theta_1)}{dn} > 0 \; \forall \; \theta_1 \in (\underline{\theta}_1, \mu]$ if private information is sufficiently precise, $\underline{\beta} < \beta < \infty$, and public information is sufficiently imprecise, $0 < \alpha < \overline{\alpha}$. Finally, we denote $\underline{\beta}_2 < \max\{\underline{\beta}_0', \underline{\beta}_1, \underline{\beta}\} < \infty$ as the maximum of the stated lower bounds on the precision of private information. The result of Proposition 3 follows. *(q.e.d.)*

B.6 Intuition: costs and benefits of a tailored signal threshold

Consider the benefit of using a tailored signal threshold. An informed investor's marginal benefit of using a higher signal threshold $\widehat{x_I}(n,\rho,\theta_1)$ is given by

$$\begin{array}{l} b\int_{-\infty}^{\theta_2^*(n,\rho,\theta_1)} g(\widehat{x_I}(n,\rho,\theta_1)|\theta_2) f(\theta_2,\rho,\theta_1) d\theta_2 \\ -l\int_{\theta_2^*(n,\rho,\theta_1)}^{+\infty} g(\widehat{x_I}(n,\rho,\theta_1)|\theta_2) f(\theta_2,\rho,\theta_1) d\theta_2, \end{array} \tag{62}$$

which is zero when evaluated at $\widehat{x_I}(n,\rho,\theta_1) = x_I^*(n,\rho,\theta_1)\ \forall\ \rho \in \{0,\rho_H\}$ by optimality. Furthermore, equation (62) decreases monotonically in $\widehat{x_I}(n,\rho,\theta_1)$:

$$\frac{dg(\widehat{x_I}(n,\rho,\theta_1)|\theta_2)}{d\widehat{x_I}(n,\rho,\theta_1)} = \begin{cases} > 0 & if\ \widehat{x_I}(n,\rho,\theta_1) < \theta_2 \\ \leq 0 & if\ \widehat{x_I}(n,\rho,\theta_1) \geq \theta_2, \end{cases} \tag{63}$$

and $lim_{\beta\to\infty} x_I^*(n,\rho,\theta_1) = \theta_2^*(n,\rho,\theta_1)\ \ \forall\ \rho \in \{0,\rho_H\}$.

When $\theta_2^*(1,0,\theta_1) > \theta_2^*(1,\rho_H,\theta_1)$, we have that $x_I^*(n,0,\theta_1) > x_U^*(n,\theta_1) > x_I^*(n,\rho_H,\theta_1)$. Thus, the marginal benefit of increasing $x_I^*(n,0,\theta_1)$ above $x_U^*(n,\theta_1)$ is

$$p\left(\begin{array}{l} b\int_{-\infty}^{\theta_2^*(n,0,\theta_1)} g(x_U^*|\theta_2) f(\theta_2) d\theta_2 \\ -l\int_{\theta_2^*(n,0,\theta_1)}^{+\infty} g(x_U^*|\theta_2) f(\theta_2) d\theta_2 \end{array}\right) > 0, \tag{64}$$

while the marginal benefit of increasing $x_{2I}^*(n,\rho_H,\theta_1)$ above $x_U^*(n,\theta_1)$ is

$$(1-p)\left(\begin{array}{l} b\int_{-\infty}^{\theta_2^*(n,\rho_H,\theta_1)} g(x_U^*|\theta_2) f(\theta_2|\rho_H,\theta_1) d\theta_2 \\ -l\int_{\theta_2^*(n,\rho_H,\theta_1)}^{+\infty} g(x_U^*|\theta_2) f(\theta_2|\rho_H,\theta_1) d\theta_2 \end{array}\right) < 0. \tag{65}$$

These expressions are best understood in terms of type-I and type-II errors. Let the null hypothesis be that there is a crisis in region 2, such that $\theta_2 < \theta_2^*$. Each of the expressions in equations (64) and (65) has two components. The first component in each equation represents the marginal benefit of attacking when a crisis occurs. (Equivalently, this is the marginal loss from not attacking when a crisis occurs (type-I error).) The second component in each equation is negative and represents the marginal cost of attacking when no crisis occurs (type-II error).

Lemma 4, together with Proposition 1, implies the following. After a crisis in region 1, we have that $\theta_2^*(n,\rho_H,\theta_1) < \theta_2^*(n,0,\theta_1)\ \forall\ n \in [0,1]$ if $\theta_1 \in (\underline{\theta}_1, \theta_1^*)$, the fundamental in region 2 is strong, private information is sufficiently precise, and public information is sufficiently imprecise. Hence, the marginal benefit of in-

creasing $x_I^*(n,0,\theta_1)$ above $x_U^*(n,\theta_1)$ is positive, because the type-I error is relatively more costly than the type-II error. By contrast, the marginal benefit of decreasing $x_I^*(n,\rho_H,\theta_1)$ below $x_U^*(n,\theta_1)$ is positive, because the type-II error is more costly. In sum, informed investors attack more aggressively upon learning that $\rho = 0$.

B.7 Proof of Proposition 4

The proof has three cases and builds on equation (17). Equation (17) is constructed from EU_I and EU_U. The expected utility of an informed investor is

$$\mathbb{E}[u(d_i = I, n)] \equiv EU_I - c \tag{66}$$

$$= -c + p\begin{pmatrix} b\int_{-\infty}^{\theta_2^*(n,0,\theta_1)} \int_{x_{i2}\le x_I^*(n,0,\theta_1)} g(x_{i2}|\theta_2)dx_{i2}f(\theta_2|0,\theta_1)d\theta_2 \\ -l\int_{\theta_2^*(n,0,\theta_1)}^{+\infty} \int_{x_{i2}\le x_I^*(n,0,\theta_1)} g(x_{i2}|\theta_2)dx_{i2}f(\theta_2|0,\theta_1)d\theta_2 \end{pmatrix}$$

$$+(1-p)\begin{pmatrix} b\int_{-\infty}^{\theta_2^*(n,\rho_H,\theta_1)} \int_{x_{i2}\le x_I^*(n,\rho_H,\theta_1)} g(x_{i2}|\theta_2)dx_{i2}f(\theta_2|\rho_H,\theta_1)d\theta_2 \\ -l\int_{\theta_2^*(n,\rho_H,\theta_1)}^{+\infty} \int_{x_{i2}\le x_I^*(n,\rho_H,\theta_1)} g(x_{i2}|\theta_2)dx_{i2}f(\theta_2|\rho_H,\theta_1)d\theta_2 \end{pmatrix}.$$

By contrast, the expected utility of an uninformed investor is

$$\mathbb{E}[u(d_i = U, n)] \equiv EU_U \tag{67}$$

$$= p\begin{pmatrix} b\int_{-\infty}^{\theta_2^*(n,0,\theta_1)} \int_{x_{i2}\le x_U^*(n,\theta_1)} g(x_{i2}|\theta_2)dx_{i2}f(\theta_2|0,\theta_1)d\theta_2 \\ -l\int_{\theta_2^*(n,0,\theta_1)}^{+\infty} \int_{x_{i2}\le x_U^*(n,\theta_1)} g(x_{i2}|\theta_2)dx_{i2}f(\theta_2|0,\theta_1)d\theta_2 \end{pmatrix}$$

$$+ (1-p)\begin{pmatrix} b\int_{-\infty}^{\theta_2^*(n,\rho_H,\theta_1)} \int_{x_{i2}\le x_U^*(n,\theta_1)} g(x_{i2}|\theta_2)dx_{i2}f(\theta_2|\rho_H,\theta_1)d\theta_2 \\ -l\int_{\theta_2^*(n,\rho_H,\theta_1)}^{+\infty} \int_{x_{i2}\le x_I^*(n,\rho_H,\theta_1)} g(x_{i2}|\theta_2)dx_{i2}f(\theta_2|\rho_H,\theta_1)d\theta_2 \end{pmatrix}.$$

First, for $\theta_1 = \underline{\theta}_1$ there are no benefits from acquiring information because $x_I^*(n,\rho,\underline{\theta}_1) = x_U^*(n,\underline{\theta}_1)\ \forall \rho$. Hence, $\overline{c}(n,\underline{\theta}_1) = 0\ \forall\, n \in [0,1]$.

Second, if $\underline{\theta}_1 < \theta_1 < \theta_1^*$, then $\theta_2^*(n,0,\theta_1) > \theta_2^*(n,\rho_H,\theta_1)$ and $x_I^*(n,0,\theta_1) > x_U^*(n,\theta_1) > x_I^*(n,\rho_H,\theta_1)$ under the sufficient conditions of Lemma 3 and Lemma 4. We will prove that $\frac{d\overline{c}(n,\theta_1)}{dn} \ge 0\ \ \forall\, \theta_1 \in (\underline{\theta}_1, \theta_1^*)$ and $\overline{c}(n,\theta_1) > 0\ \ \forall\, \theta_1 \in (\underline{\theta}_1, \theta_1^*)$.

An increase in the proportion of informed investors is associated with a (weak) increase in both $\theta_2^*(0)$ and $x_2^*(0)$ as well as a (weak) decrease in both $\theta_2^*(\rho_H)$ and $x_2^*(\rho_H)$. Furthermore, $x_U^*(n,\theta_1)$ is unaffected. An increase in n leads to a relative increase in the benefit component in the first summand of equation (17), and a rel-

ative increase in the loss component in the second summand. For this reason, the left-hand side of equation (17) increases in n. Thus, $\frac{d\bar{c}(n,\theta_1)}{dn} \geq 0 \ \forall \ \theta_1 \in (\underline{\theta}_1, \theta_1^*)$.

It remains to consider the case of $\theta_1 < \underline{\theta}_1$. Here, we have $\theta_2^*(1,0,\theta_1) < \theta_2^*(1,\rho_H,\theta_1)$ and $\theta_2^*(1,0,\theta_1) \leq \theta_2^*(n,\rho,\theta_1) \leq \theta_2^*(1,\rho_H,\theta_1) \ \forall \rho \in \{0,1\}$. Hence, $x_I^*(n,0,\theta_1) < x_U^*(n,\theta_1) < x_I^*(n,\rho_H,\theta_1)$. We will prove that $\frac{d\bar{c}(n,\theta_1)}{dn} \geq 0 \ \forall \ \theta_1 < \underline{\theta}_1$ and $\bar{c}(n,\theta_1) > 0 \ \forall \ \theta_1 < \underline{\theta}_1$.

Again, it is optimal to purchase information if the differential expected payoff is positive. Given that $\theta_2^*(1,0,\theta_1) < \theta_2^*(1,\rho_H,\theta_1)$, the first two summands in (17) are strictly positive and, thus, $\bar{c}(n,\theta_1) > 0 \ \forall \ \theta_1 < \underline{\theta}_1$. Furthermore, an increase in n is associated with a (weak) decrease in $\theta_2^*(0)$ and $x_2^*(0)$, and a (weak) increase in $\theta_2^*(\rho_H)$ and $x_2^*(\rho_H)$. For this reason, an increase in n leads to a relative increase in the loss component in the first summand of equation (17) and a relative increase in the benefit component in the second summand. As a result, we have that the left-hand side of equation (17) increases in n. Thus, $\frac{d\bar{c}(n,\theta_1)}{dn} \geq 0 \ \forall \ \theta_1 < \underline{\theta}_1$, which concludes the proof. *(q.e.d.)*

B.8 Proof of Proposition 5

The result follows from Proposition 4 in combination with Proposition 2. From Proposition 4 there exists a strictly positive cost level, $c < \bar{c}(0,\theta_1)$, such that information acquisition occurs for all $\theta_1 \neq \underline{\theta}_1$, i.e. $n^* = 1$. Hence, there exists a unique pure-strategy PBE where the wake-up-call contagion effect arises if private signals are sufficiently precise, $\beta > \max\{\underline{\beta}_2, \underline{\beta}_4\}$, and the public signal sufficiently imprecise, $\alpha < \overline{\alpha}$. *(q.e.d.)*